Intrinsic MHD turbulence in magnetically confined fusion plasmas

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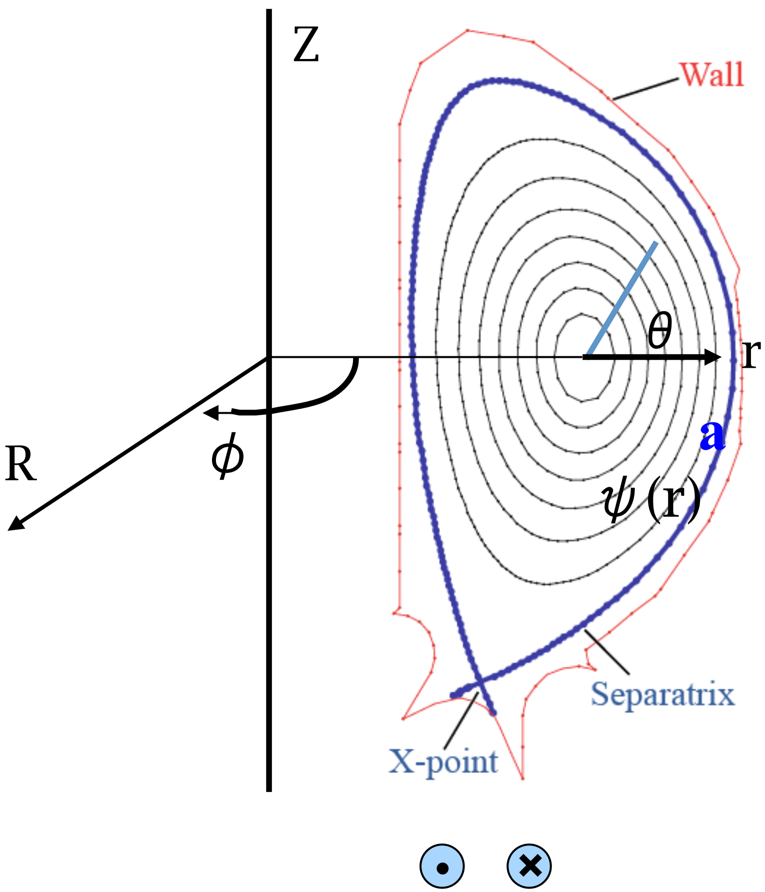
Topics

MHD "turbulence" is electromagnetic, not the mostly electrostatic turbulence that drives standard transport

- Toroidal magnetic field + Nonlinearity
  - Small transverse perturbations produce Hamiltonian magnetic stochasticity
- MHD plasma dynamics provides required $\perp\mathbf{B}$ motion
  - Important instabilities in interior and edge of a fusion plasmas
- MHD instabilities can also generate stochasticity by other mechanisms
- Implications – theory

Toroidal plasma



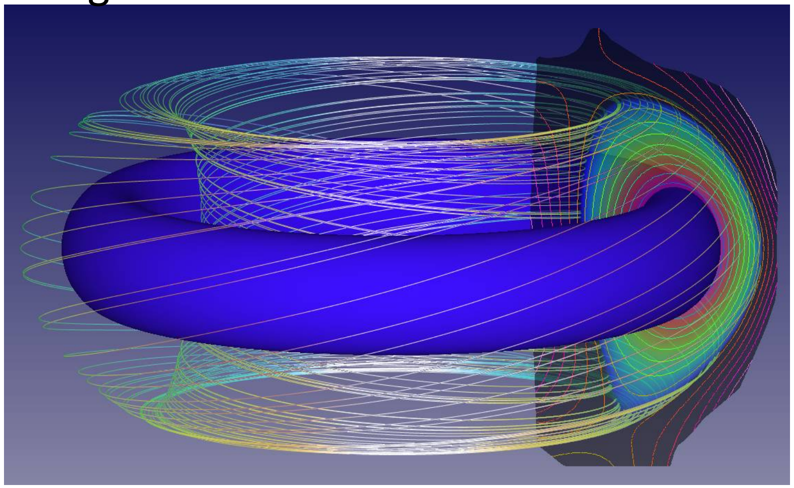
B_T I_p

$R = R_o + r\cos\theta$

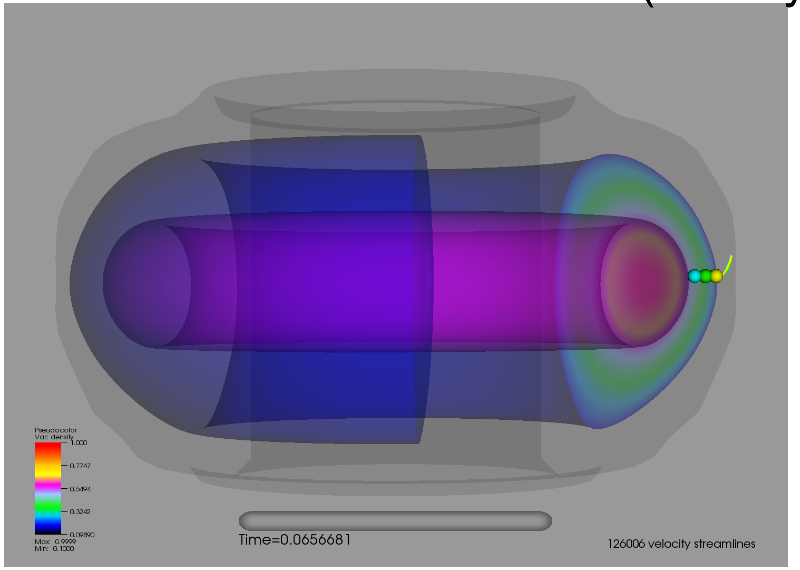
Axisymmetry:

toroidal harmonic n is an eigenvalue

Magnetic field lines on nested surfaces



Plasma follows flux surfaces (density)



Toroidal and periodic-cylinder magnetic fields form Hamiltonian systems

- $\nabla \cdot \mathbf{B}=0$ reduces the dimensionality of the field by 1
  - Axisymmetric toroidal field (dipole, torus, periodic cylinder) is a 1 degree of freedom Hamiltonian system [V.I. Arnold, 1960's]
  - Non-axisymmetric toroidal fields are 2 d.o.f. Hamiltonian systems [A. Boozer, Phys. Plasmas 1982-83]
- Hamiltonian dynamics of 2 d.o.f systems studied extensively in late 1970's and 1980's, with aid of early numerical simulations; theoretical insights
  - Break-up of system of axisymmetric nested torii with increasing non-axisymmetric perturbation size follows general rules (KAM theorem)
  - Hyperbolic saddle points (X-point of islands) can create "homoclinic" tangle
- Nonlinearity is essential!

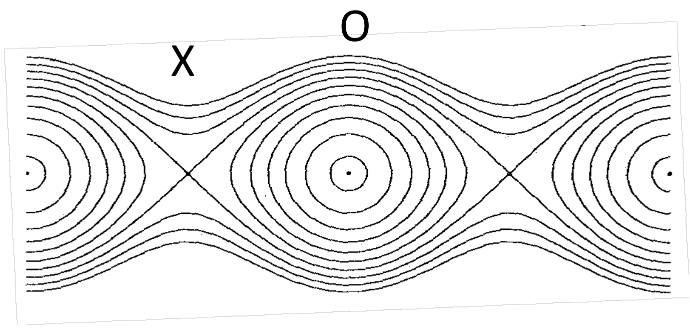
KAM theorem and magnetic islands

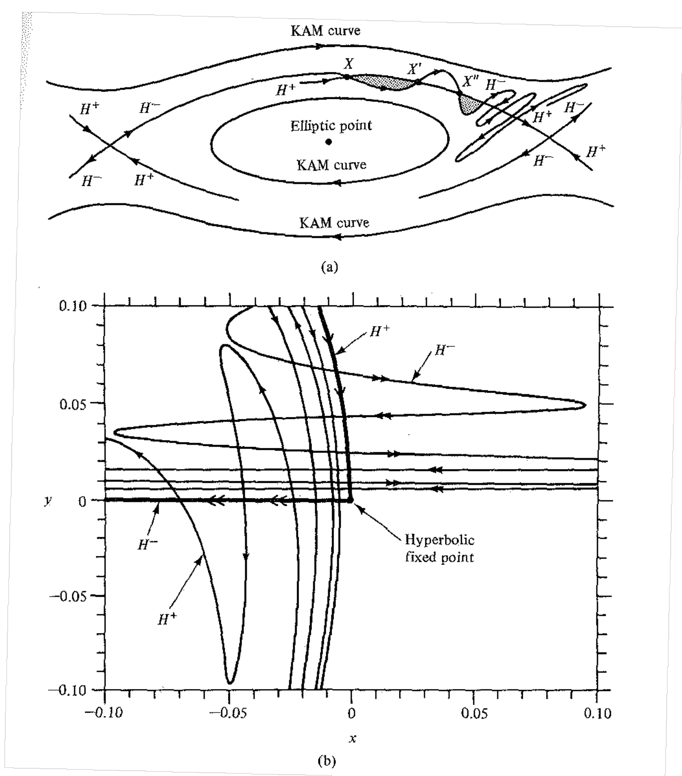
- Kolmogorov-Arnold-Moser theorem describes the nonlinear breakup of a system of nested torii under perturbation [eg, Lichtenberg and Lieberman, Regular and Chaotic Dynamics, Springer 2nd ed (1992)]
  - Lowest rational surfaces (winding number =n/m) break up up first into magnetic islands (X and O-points)
  - As perturbation grows, more islands form; Edges of the main islands break up in similar fashion into higher order island chains;
  - Main island chains begin to overlap.
  - Longest lasting surface has the ‘most irrational’ number, golden mean $(\sqrt{5} - 1)/2$
- Description is qualitative: still difficult to quantify island width, degree of overlap in real plasma

Stellarator/helical plasma "soft beta" limit

- Helical plasmas such as stellarators do not have a, MHD-instability-driven limit on maximum pressure ($\beta=2p/B^2$), but a "soft" limit as the transport losses increase
- Related to KAM breakup of flux surfaces
  - Helical magnetic imbalances from external coils typically drive islands at lowest order rational $q=m/n$ magnetic surfaces; Mismatch and island chain width/stochasticity typically increase with pressure, reducing the possible pressure gradient; Plasma reaches a pressure limit at given heating power
- Axisymmetric plasmas: β or ∇p grows until a strong MHD instability is triggered and confinement is abruptly lost

Magnetic tangle: Hamiltonian system





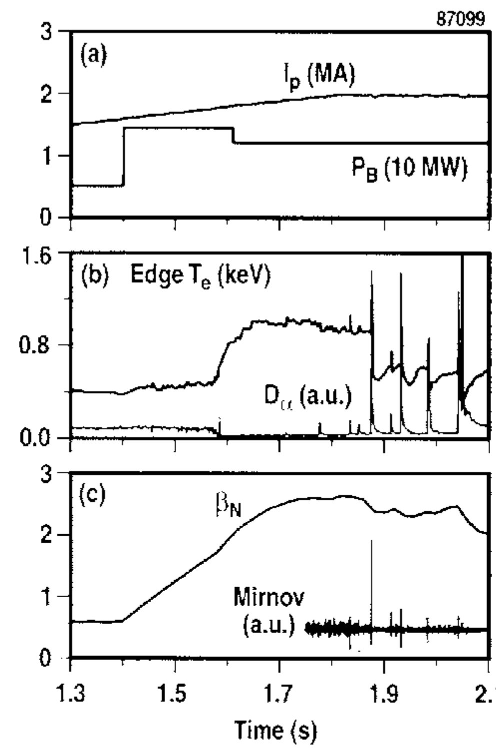
- 1D pendulum in (x,v) phase space is simple Hamiltonian system, periodic in x: $d^2x/dt^2 + \sin 2\pi x = 0$, v=dx/dt
  - The separatrix through the X-points divides a swinging motion (inside, around the O-point) from complete rotation (outside).
- Transverse perturbation of the separatrix, e.g., by a forcing term at different frequency, causes the separatrix surface (a manifold) to split into 2 different asymptotic limits, with complicated behavior near an X-point.
- Trajectories formed by the extended loops on the two sides of the X-point intersect many times and become chaotic. Similar trajectory splitting occurs at each new X-point.

from Lichtenberg and Lieberman, Regular and Chaotic Dynamics (1992)

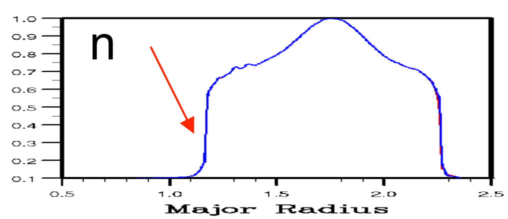
Magnetically Confined Fusion Plasma

- Magnetically confined fusion plasmas (axisymmetric) typically have D-shaped cross sections, with one or two magnetic X-points on the plasma boundary.
- X-point was intended to help control losses by channeling them along outer legs to special wall regions (divertors)

H-mode operation (ASDEX 1982): D-shape gives good overall confinement of particles and energy
- Steep edge pressure gradient, combined with good plasma confinement in the edge region
- A minimum heating power is required to reach H-mode
- But, edge pressure gradient drives a periodic edge instability - large Edge Localized Mode (ELM) or other, more benign oscillations

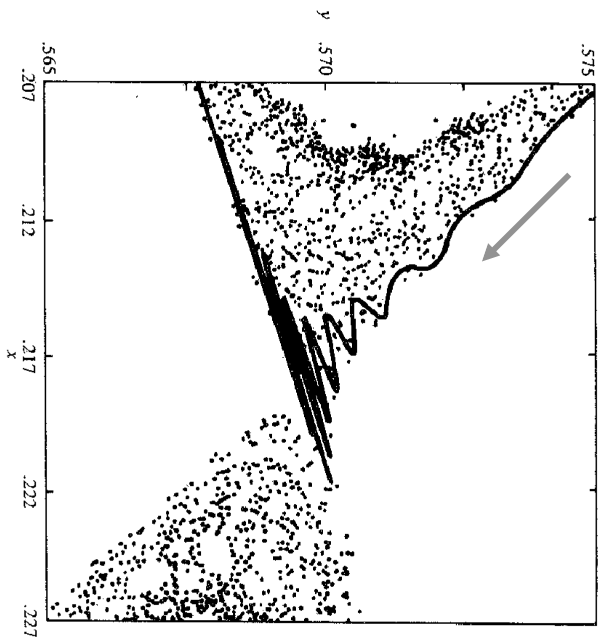


Physics of the plasma
edge is still not understood



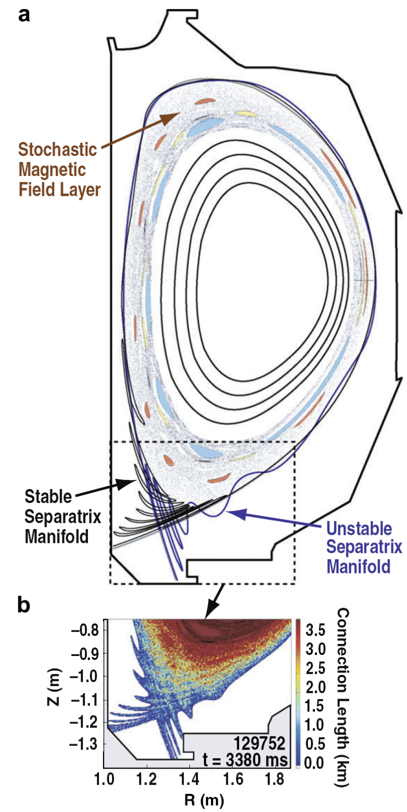
Toroidal magnetic field is a Hamiltonian system
X-point behaves like a hyperbolic saddle point

Transverse perturbation of separatrix with X-point produces asymptotic field line splitting → "homoclinic" tangle → chaotic field



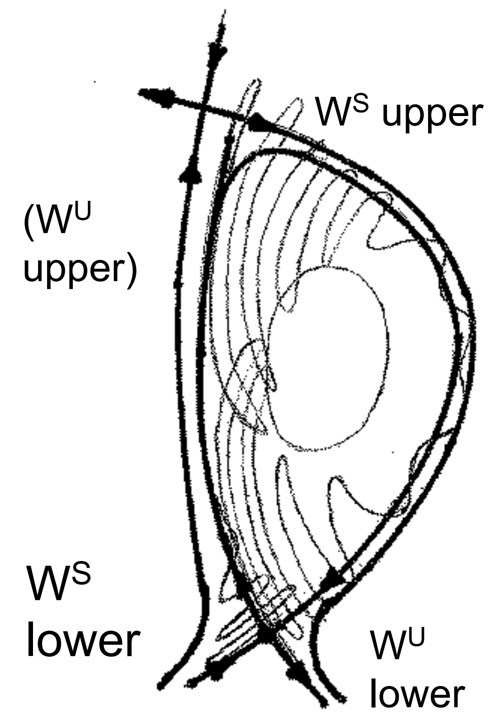
Exact Hamiltonian (perturbed pendulum)

Lichtenberg & Lieberman, Regular and Chaotic Dynamics (1992).



DIII-D with RMP vacuum field: $\delta B/B \sim$ few x 10^{-4}

T Evans, J Nuc Mat 2007



ELM (schematic)

Sugiyama, PoP 2010

MHD plasma instabilities produce cross-field perturbations

- Plasma MHD describes zero Larmor radius, short mean free path limit of plasma dynamics; electromagnetic
  - For high temperature fusion plasmas, neglects long mean free path particle dynamics along **B** --- MHD instabilities primarily move across **B**
- MHD instabilities $\perp$**B** provide a natural seed perturbation for Hamiltonian magnetic field break-up
  - MHD plasma instabilities interact nonlinearly with the stochastic field; field can shape the instability (ELMs/edge, plasma disruptions)
  - MHD motion not tied to field lines: interchange instability
- MHD instabilities can also generate stochasticity through other mechanisms – nonlinear mode coupling in toroidal and/or poloidal directions

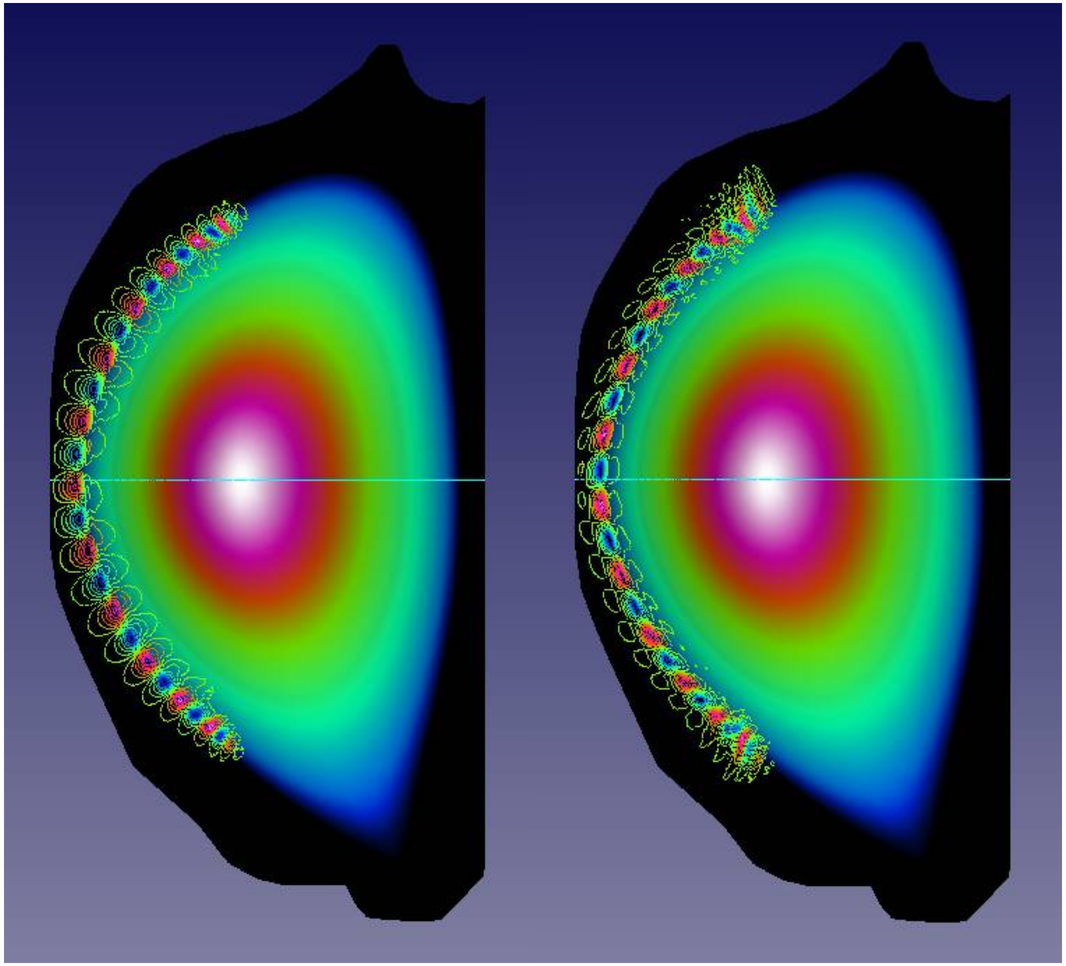
Edge Localized Modes (ELMs)

- Edge instabilities associated with the magnetic "X-points" are important for fusion plasmas
  - Steep gradient of pressure (n, T) near the edge of the plasma in H-mode drives "ballooning" or "peeling" modes that can expel large amounts of plasma in very short times (few 10's μs)
  - In fusion burning plasmas, ELMs can transfer large amounts of energy to local regions on the material walls – severe damage (next generation ITER experiment can tolerate only a very limited number of full strength ELMs)
- Methods to stabilize ELMs with small applied non-axisymmetric fields demonstrated experimentally
  - Creates stochastic field -- stabilization not yet understood

Vacuum region is important to instabilities

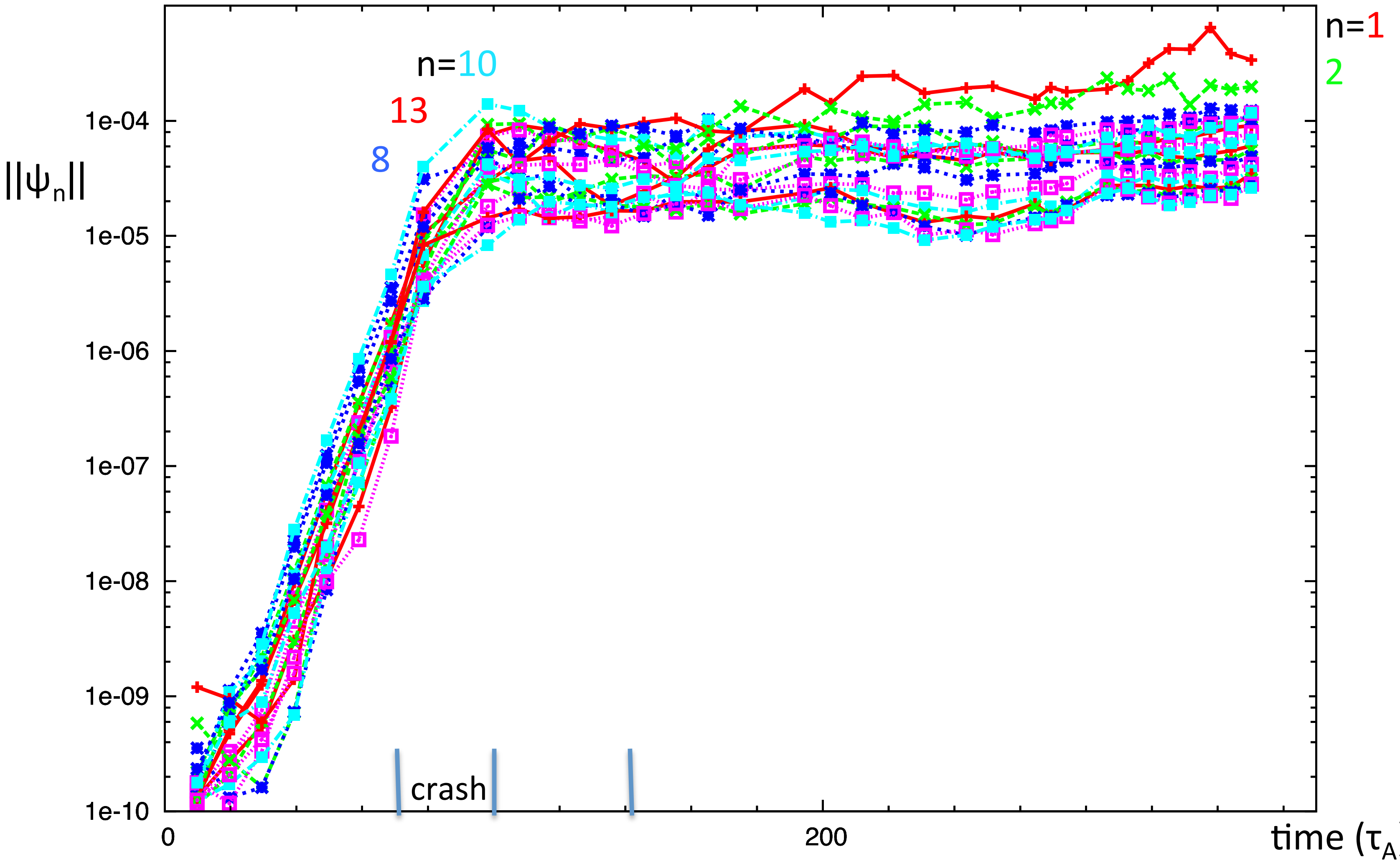
- Resistive MHD vacuum:
  high-η, low J, low density, zero T plasma
- Freely moving plasma boundary – mode disturbs plasma edge
- Magnetic perturbation extends to wall; η_{wall} effect
- Growth rate depends on vacuum resistivity η_v
  (Ferraro PoP12). $S_V=10^4$ at T=30-100 eV (M3D uses $S_V=10^3$))
- Growth rates of interior instabilities also depend on vacuum region (eg 1/1 mode)

Linear ballooning eigenmode, n=20 DIII-D ELM



ψ U

ELM spectrum – strong nonlinear mode coupling



- Poloidal magnetic flux grows exponentially to initial density ballooning "crash"
- Started with all n=1-23 harmonics; n=10,8,13 grow to crash; n=1,2 late (volume integrated norm $||f_n|| = (\int d^3x\ |f_n{}^2| \ / \int d^3x)^{1/2}$, n=1-20 shown)

Nonlinear mode coupling influences ELM structure.

Stronger MHD mode coupling in spherical torus NSTX compared to DIII-D tokamak leads to differences in ELM filaments that resemble experimental observations.

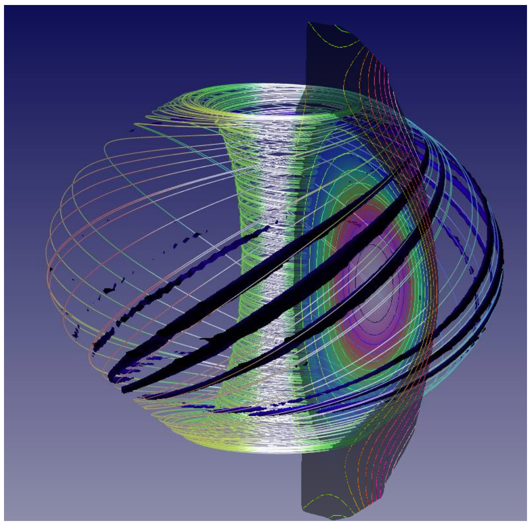
NSTX 129015

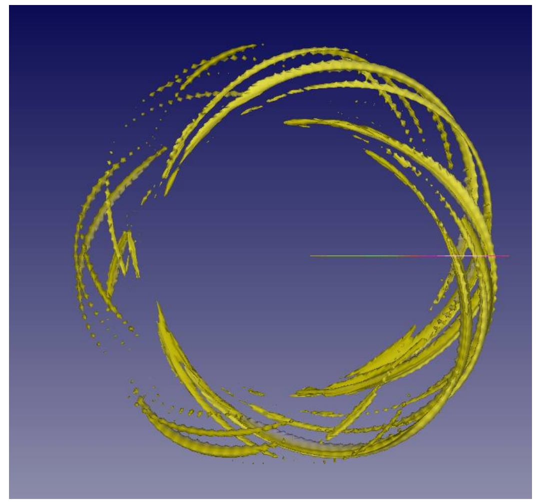
$\tilde{n}$ on B field lines

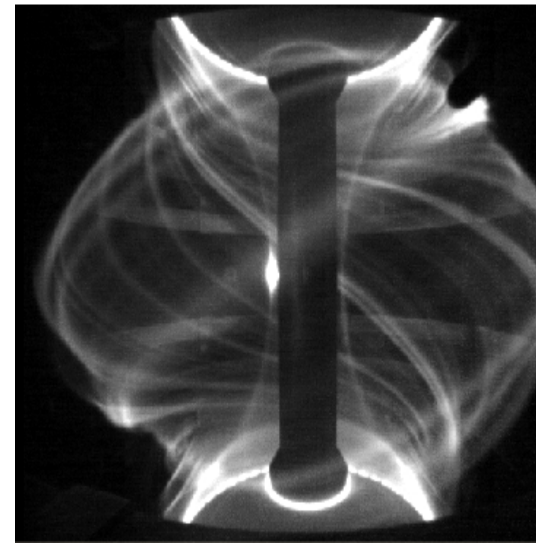
$\tilde{\psi}$ top view: low-n

experiment: MAST

NSTX





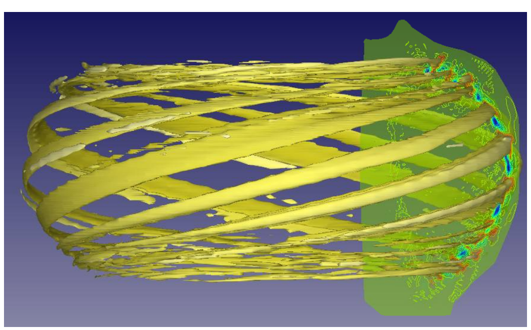


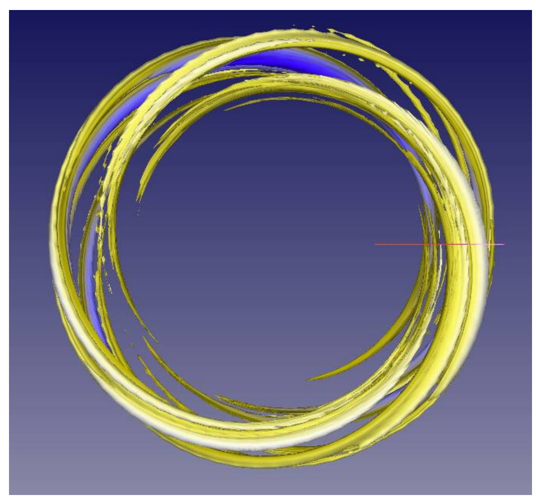
DIII-D 126006

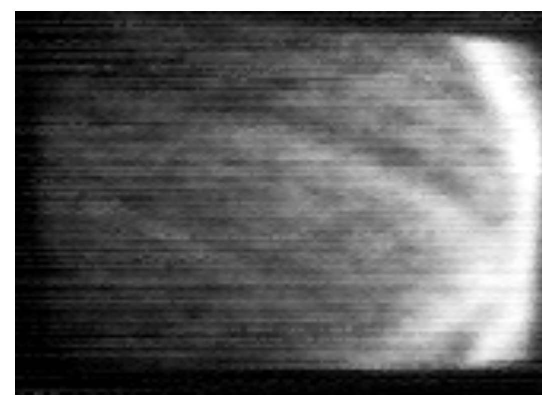
$\tilde{n}$

experiment: DIII-D

DIII-D







dominant n=10,13

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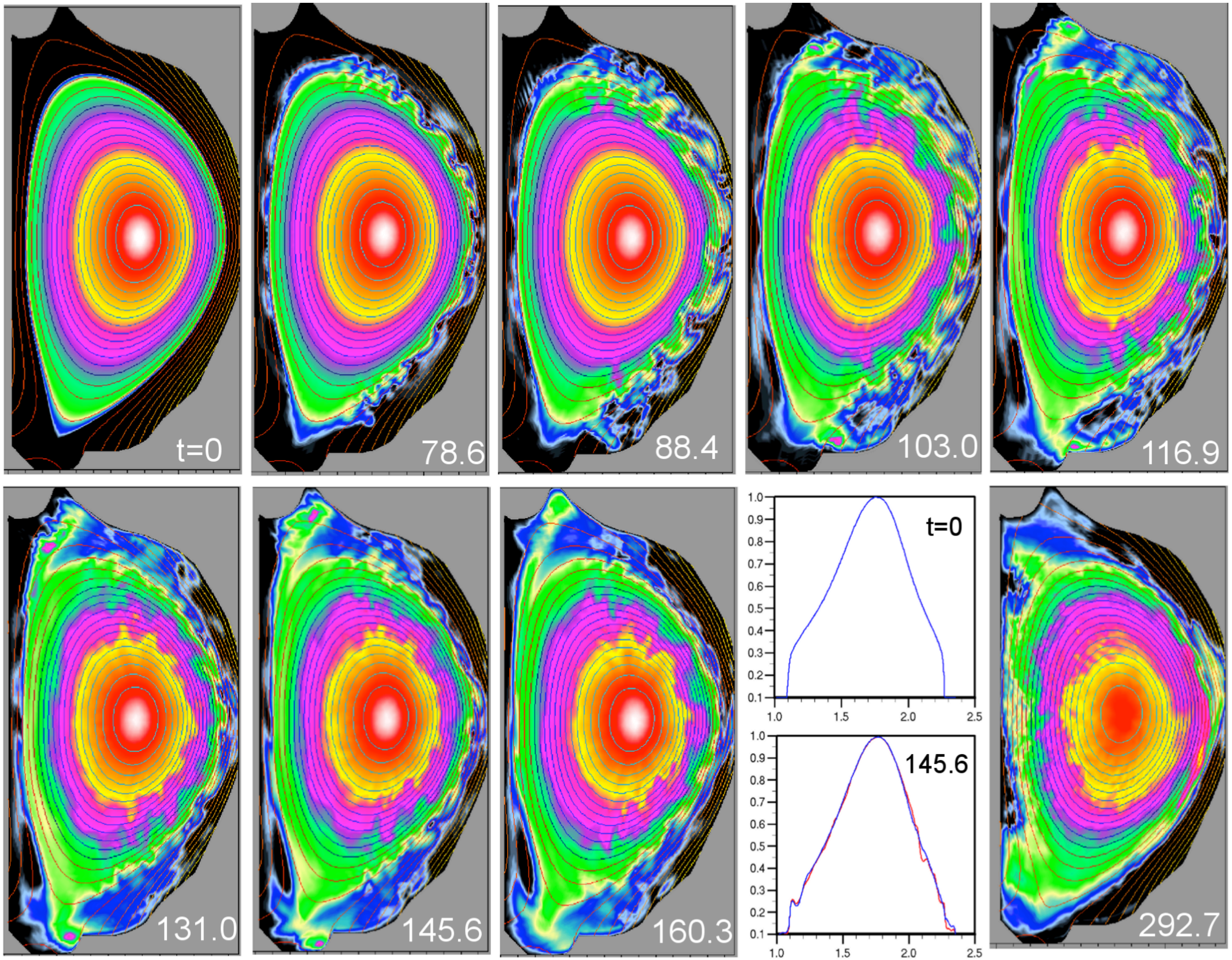
ELM nonlinear evolution: density (DIII-D)



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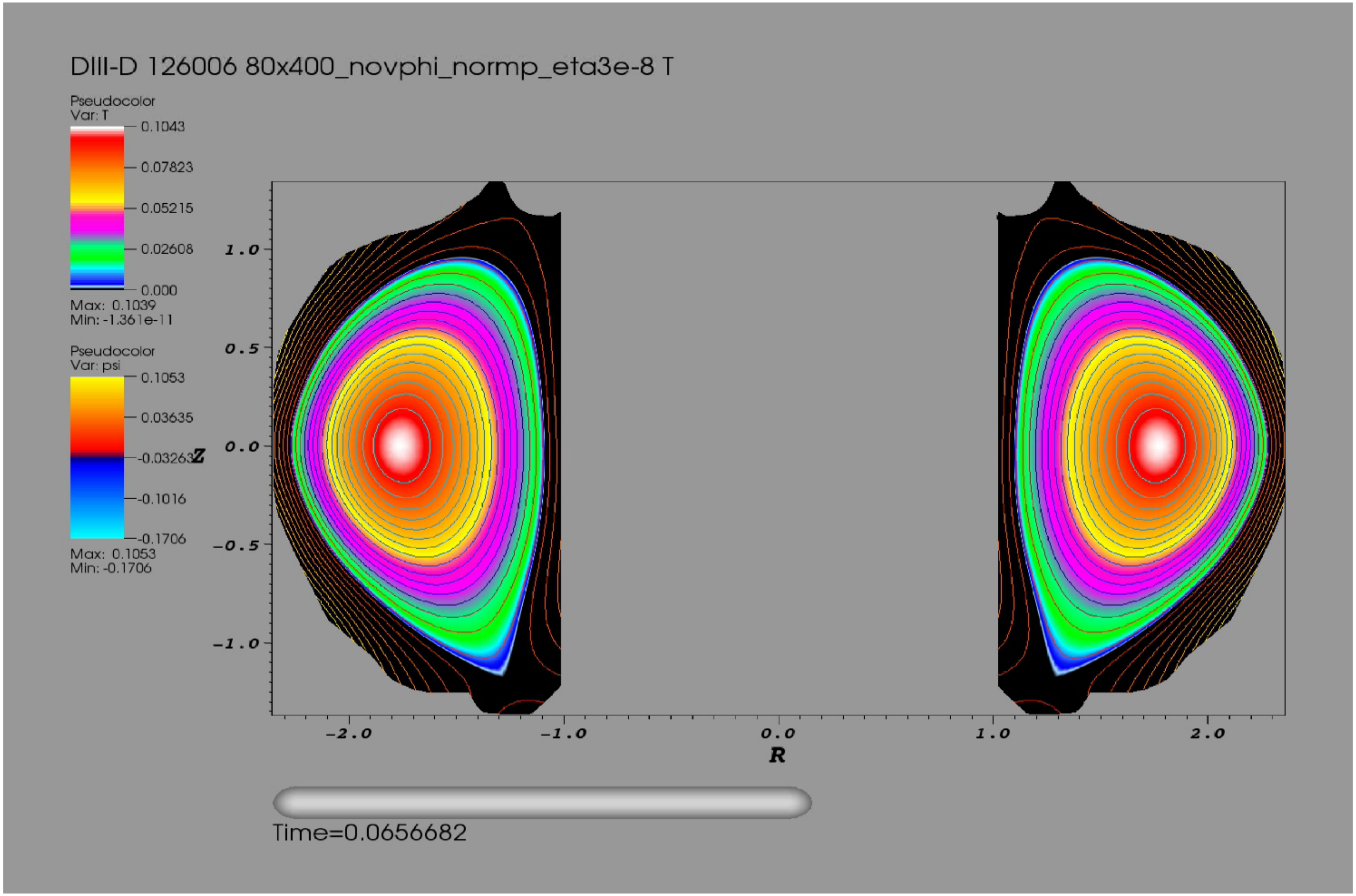
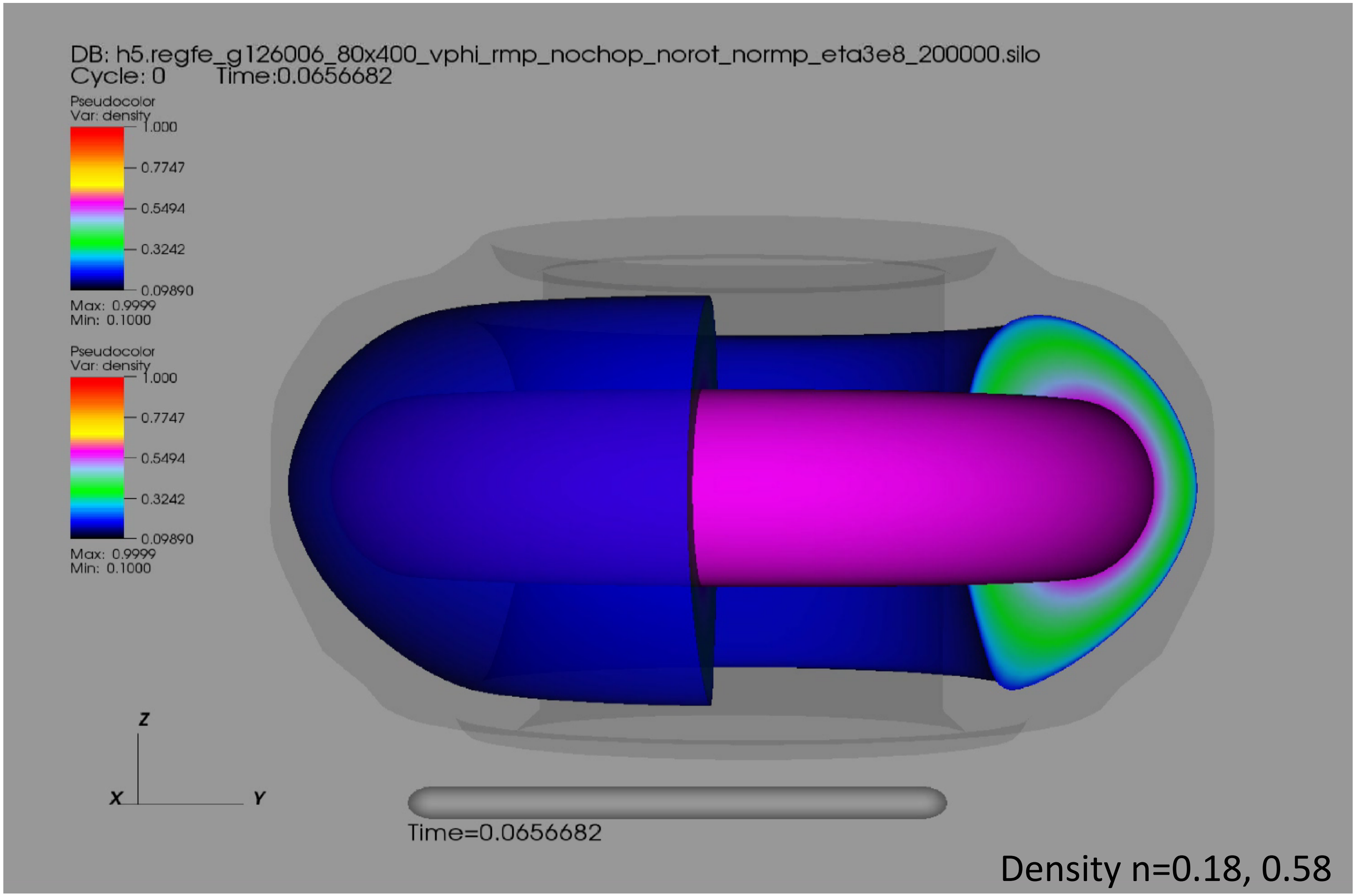
ELM: temperature (movie)



ELM generates nonlinear plasma rotation (movie)



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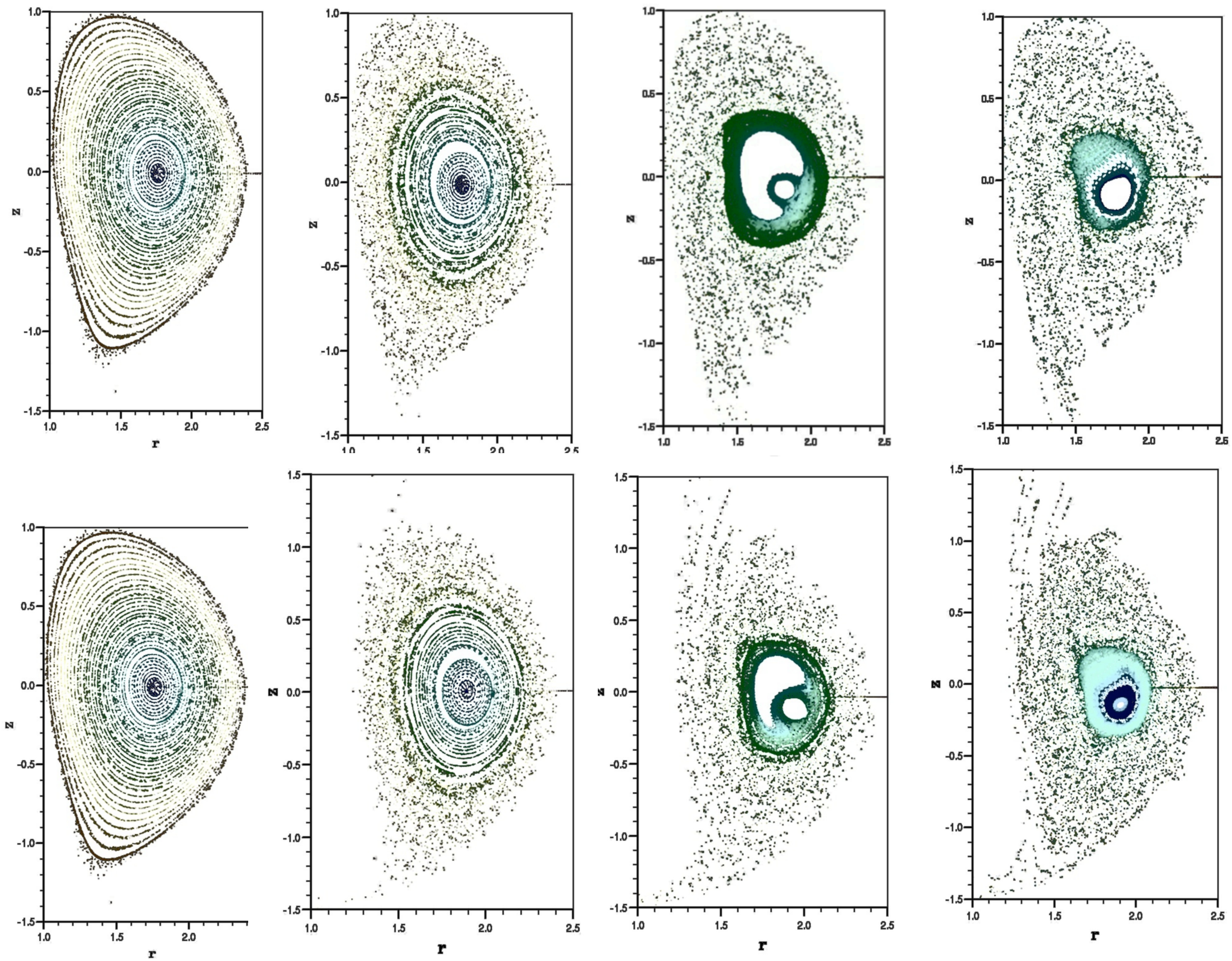
Magnetic tangle. Field lines have characteristic features of 'stable' and 'unstable' homoclinic manifolds (enlarged at high η)

+ϕ

lower X-pt
unstable

-ϕ

lower X-pt
stable
(upper X-pt
unstable)



1196090 Large ELM, large η (S=10^6)

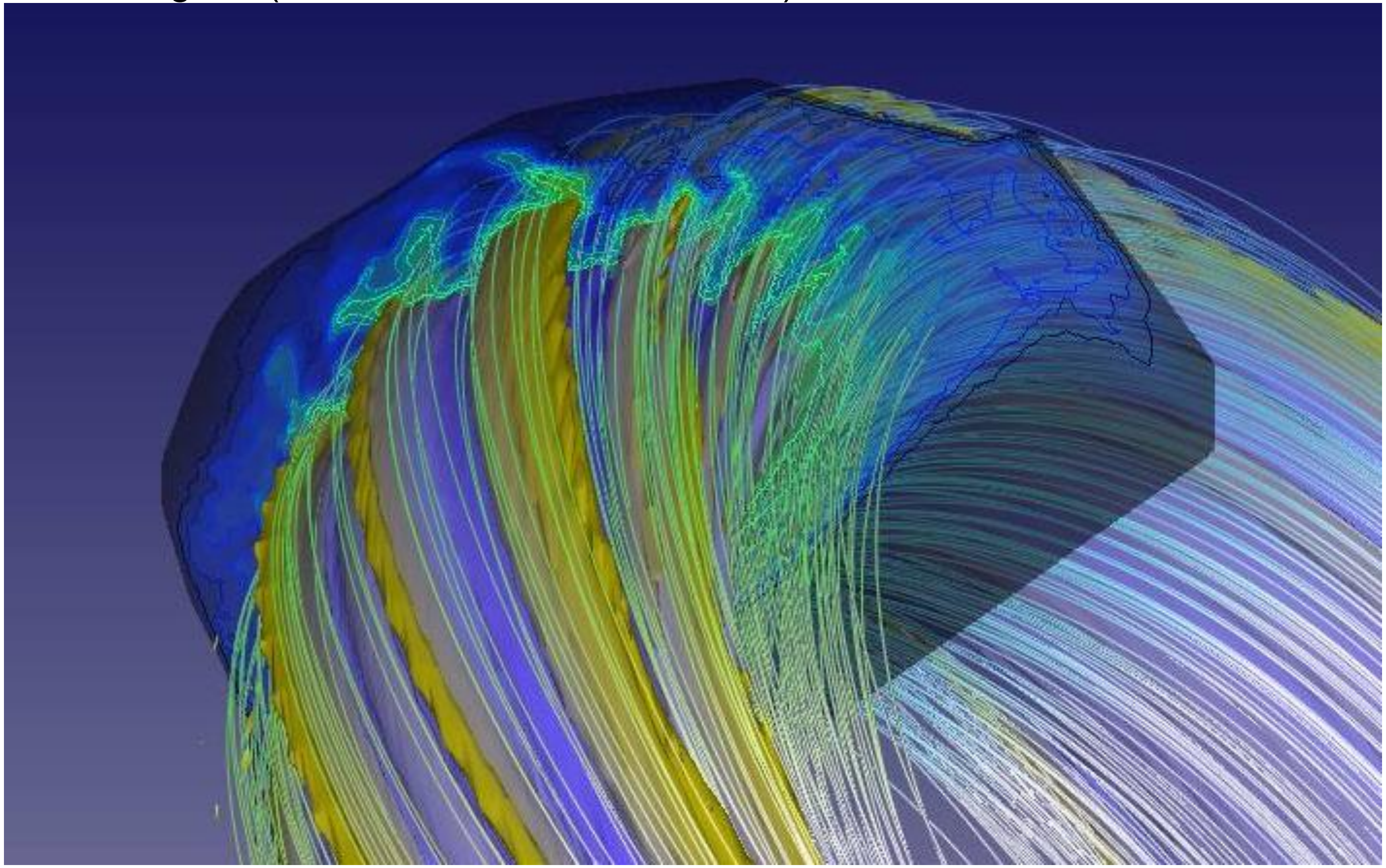
Magnetic field retains structure, despite apparent chaos.

Field lines are (1) mostly helical, similar to equilibrium,

(2) most confined for many toroidal transits, then lost from near X-points,

(3) approximately follow temperature contours, with Δr excursions.

Temperature surface colored by values of ψ. Single field line followed in $+B_\varphi$ direction. Tilted to show bottom X-region (DIII-D 119690, different case).

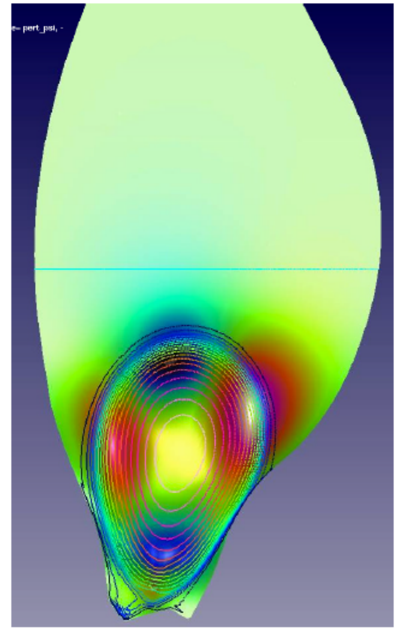


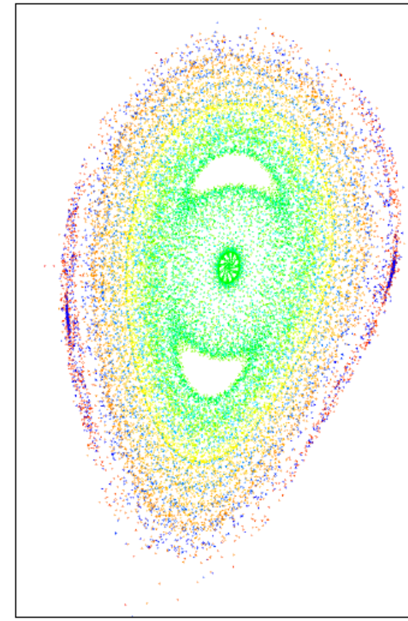
Vertical Disruption Event (VDE)

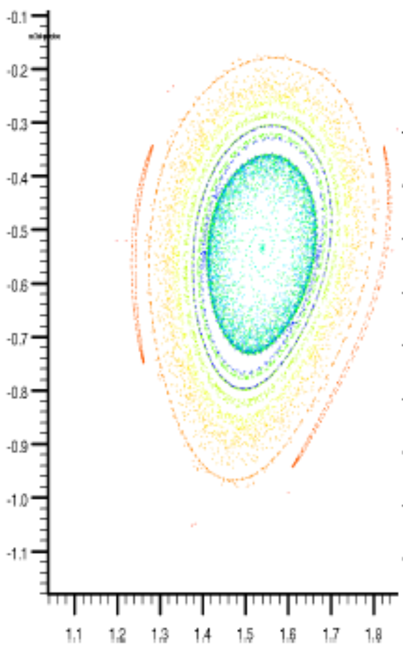
- Toroidal plasmas are unstable to vertical displacement
  - Once control is lost, a growing instability allows entire plasma to move up/downwards to the wall, deposit energy and current
  - Interaction of wall and plasma resistivity (initially low)
- Vertical displacement triggers a driven low m,n magnetic island at resonant interior magnetic surface (e.g., 2/1), which can grow very large and stochasticize the field over entire central region $q<2$
  - Once plasma loses enough edge current to bring the 2/1 island into contact with the wall, an ideal MHD instability is triggered that leads to rapid loss of the rest of the plasma
  - More dangerous than ELM/edge instability

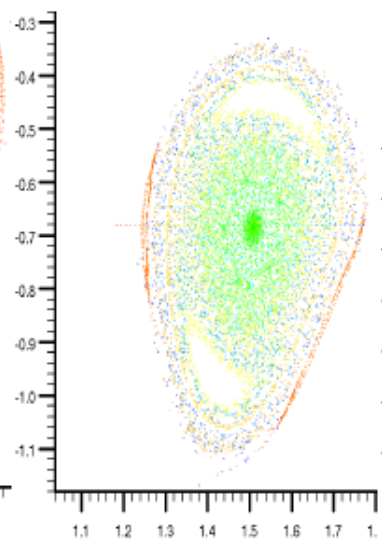
VDE magnetic stochasticity

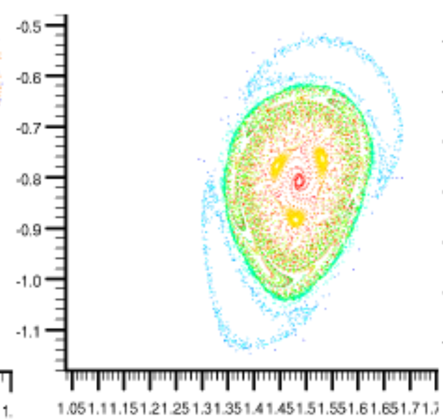
- Strong stochasticity is generated as plasma moves vertically
- Pressure approximately constant inside q<2 and 2/1 island, lower outside

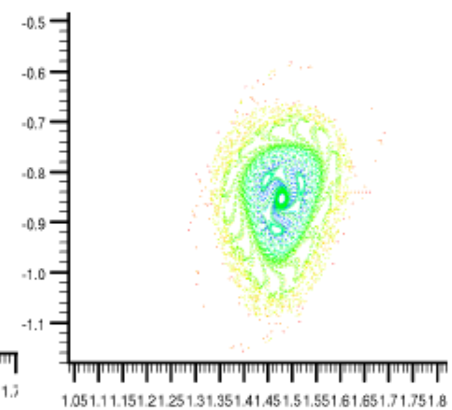












R Pacagnella, Padua

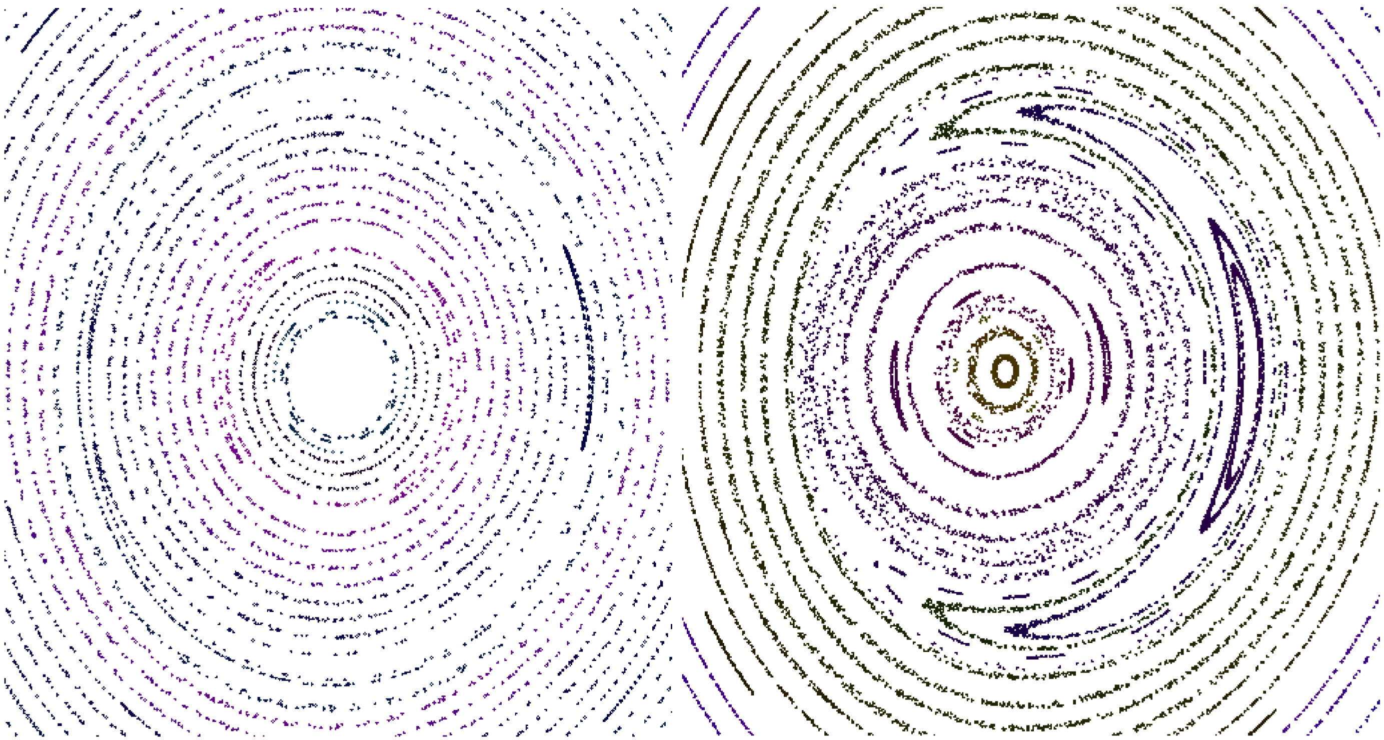
MHD instabilities can generate stochasticity: m=1,n=1 internal kink (sawtooth crash)

- 1/1 instabilities that produce a coherent helical displacement of the central plasma inside $q<1/1$are common place in fusion plasmas – full growth of the kink produces a "sawtooth" crash that flattens T and n over $q<1$, expelling some
- 1/1 mode analytical theory is a paradigm of toroidal instability (first true toroidal theory of linear ideal MHD mode, M.N. Bussac, et al, Phys Rev Lett (1975))
- Nonlinear evolution of 1/1 mode and sawtooth studied by MHD numerical simulation, but poorly understood theoretically, due to complexity
  - Theory: inverse aspect ratio expansion, $\varepsilon=r_{q=1}/R_o << 1$

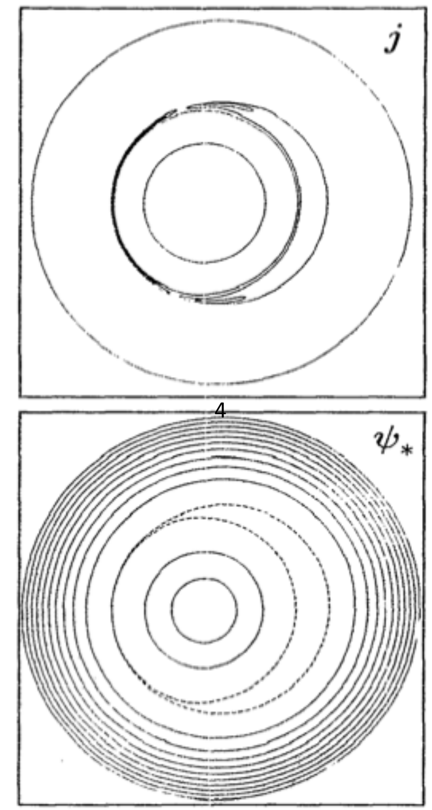
New results – fast sawtooth crash at low (realistic) resistivity

- Reduced MHD produces Sweet-Parker-like reconnection in 1/1 mode, but island width $W \sim \eta t^2$ is too slow to explain observed high temperature sawtooth crash [Waelbrock 1989; Biskamp 1991]
  - Non-MHD nonlinear electron effects (two-fluid $\nabla_{\|} p_e$ or e-inertia, etc) broadens Sweet-Parker layer and speeds up crash (slab and torus)
- Compressibility ($\nabla \cdot v$) changes MHD linear 1/1 mode in a torus
  - Solution is several orders higher order in ε than incompressible mode
- Nonlinear analog: at low resistivity, the small mode growth rate leads to initial very small $\partial \mathbf{v}_{\perp}/\partial t$ compared to $\mathbf{J} \times \mathbf{B} - \nabla p$ in momentum equation → higher order aspect ratio terms
  - m>1,n=1 terms prevent formation of a Sweet-Parker-like reconnection layer (narrow "Y-line" layer that extends over ±0.8π/2 in poloidal angle), and produces a more open "X-point")
  - Fast crash, with fast onset at a critical amplitude, linked to growth of higher toroidal harmonics (different from space fast reconnection)

Full MHD reconnection is not thin Sweet-Parker layer



MHD



RMHD (Biskamp '91)

- X-shape appears as soon as island visible (case; helical-density-triggered crash in Alcator C-Mod-like equilibrium, $S=10^8$, at $t=305$)
- Later stage has X-shape with ring of stochastic field around central core (natural sawtooth $S=10^6$, island width $W/r_1 \simeq 1/2$, $t=579$).

Full MHD vs. Large aspect ratio vs. RMHD

Perpendicular (to ϕ) momentum equation at $q = 1$ gives radial inflow, poloidal outflow of reconnection layer. Neglecting viscosity,

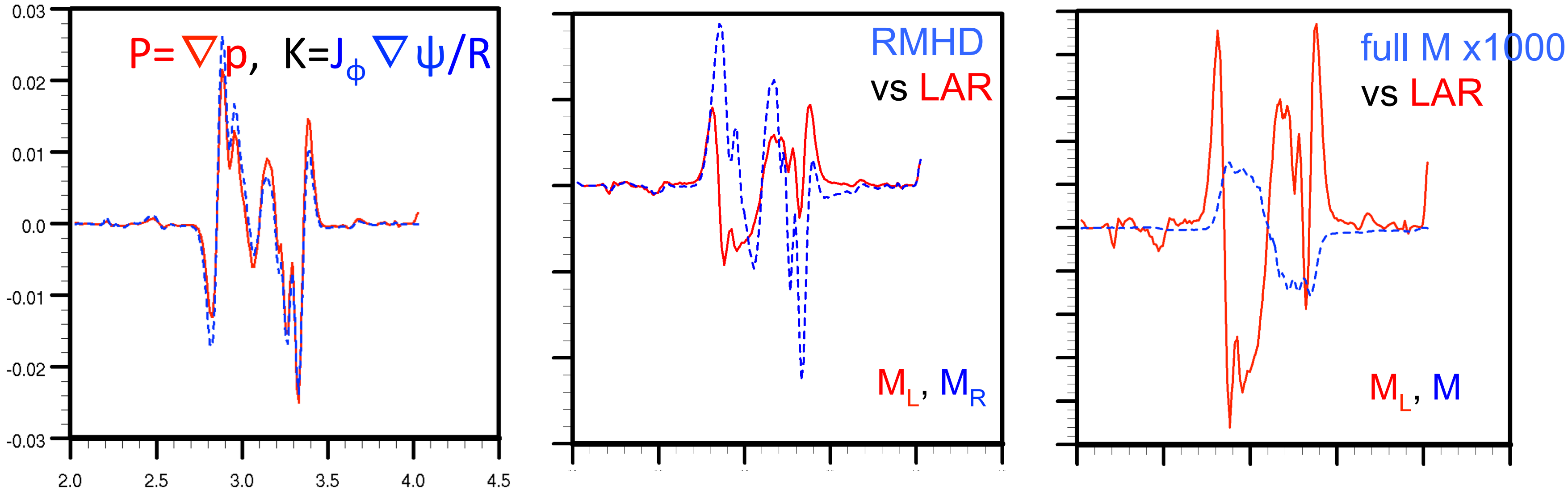
$$\rho(\partial \mathbf{v}_\perp/\partial t) = -\rho(\mathbf{v}\cdot\nabla)\mathbf{v}_\perp + (\rho v_\phi^2/R)\hat{\mathbf{R}} + (\mathbf{J}\times\mathbf{B} - \nabla p)_\perp \equiv \text{-}\mathbf{M}.$$

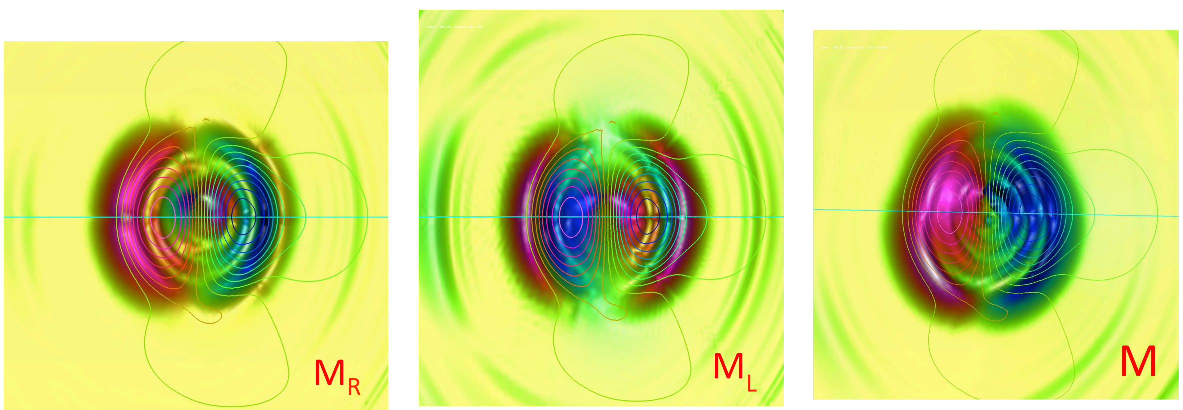
$$\begin{array}{rrcl} \text{RMHD (Biskamp ‘99)} & \mathbf{M}_R & = & \nabla_\perp(p + \rho v_\perp^2/2 + B^2/2) \\ \text{Large Aspect Ratio} & \mathbf{M}_L & = & \mathbf{P} - \mathbf{K} \\ & \mathbf{P} & = & \nabla_\perp(p + I^2/2) + \rho\nabla_\perp(v_\perp^2/2) \\ & \mathbf{K} & = & (J_\phi/R)\nabla_\perp\psi \end{array}$$

Full MHD (radial component):

$$\begin{array}{rcll} M_r & = & p' + \rho(v_\perp^2/2)' + (I^2/2)' - (J_\phi/R)(\psi' - F‘) & \text{M}_\text{L} \\ & & +((R_o/R)^2 - 1)(I^2/2)' - (R_o/R)^2(I/R)\left((\partial\psi/\partial\phi)‘ + (\partial F/\partial\phi)'\right) & \text{M}_\text{I} \\ & & +\rho(R/R_o)w((R/R_o)U' - \chi‘) - \rho(v_\phi^2/R)(\nabla r\cdot\nabla R) & \text{M}_\text{V} \\ & & +\rho(v_\phi/R)\left((\partial\chi/\partial\phi)' + (R/R_o)(\partial U/\partial\phi)‘\right) & \\ & & -\mu\left(\nabla^2\chi' + \nabla^2 U‘\right). & \end{array}$$

Perpendicular momentum terms $M \equiv \partial v_r/\partial t$

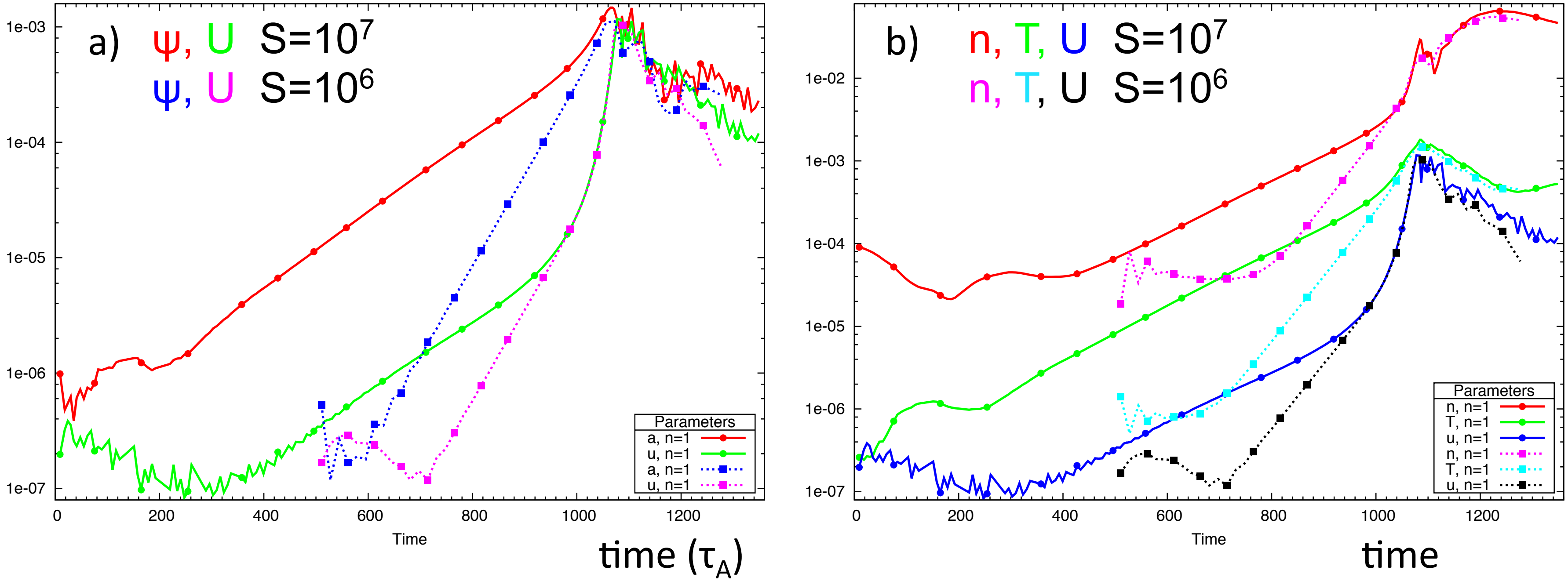




M's over central region have different forms (Contour lines show $\tilde{\psi}$. Kink moves to right).

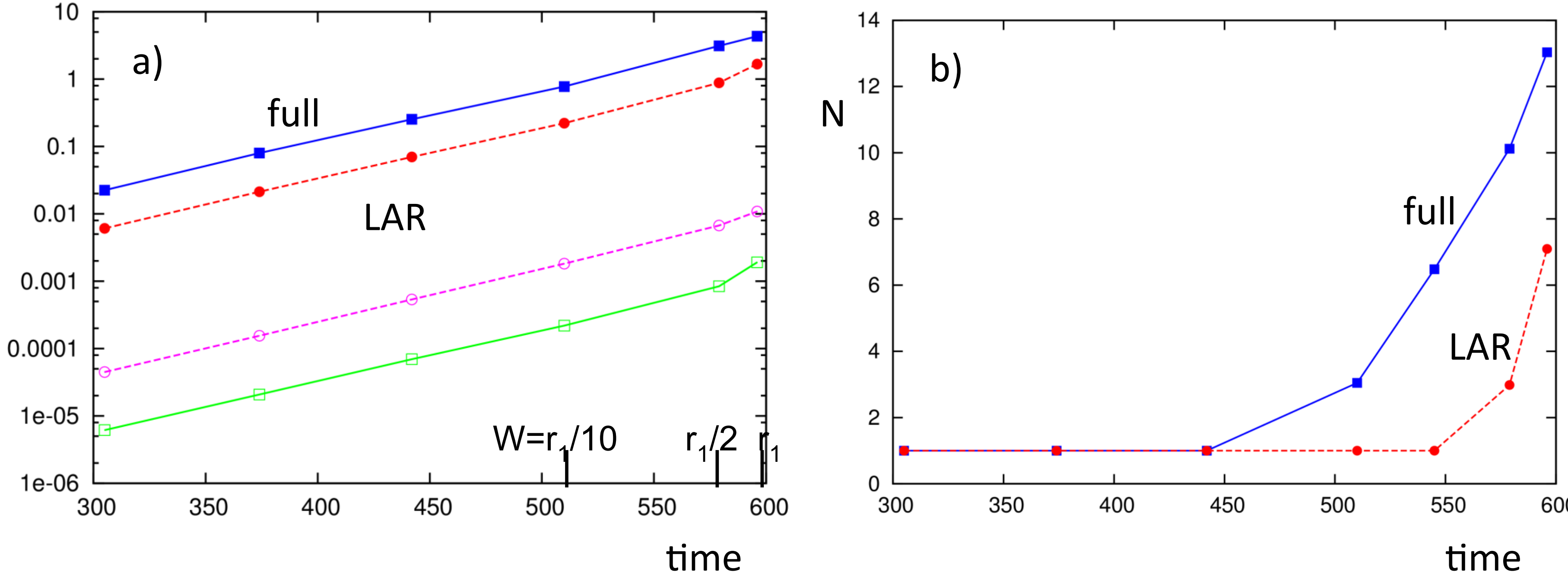
- Momentum terms P, K, M_L, M_R, M at early $t=442$, no or very small island.
- LAR $M_L = P - K \approx (1/5)P$ cancels well locally, but wrong shape
- RMHD $M_{R\approx}$ has poor cancellation, but 1/1 Sweet-Parker form
- Full M *is very small*; has m=2 inside q<1 and q=1

Fast crash is similar for S=10^6-10^8



- Natural sawteeth with different initial conditions (density perturbations) still have very similar fast crash despite different early growth rates (γ =0.018, 0.0071). Similar critical amplitude U.
- a) S=10^7 (red ψ, green U) and S=10^6 (blue/pink) fast crash time histories are similar. S=10^6 case is rigidly displaced in time to overlay fast crash in U. Harmonic n=1.
- b) n, T, U for harmonic n=1.

Full MHD develops more higher harmonics faster



- a) Ratio of amplitude in the n≥2 harmonics to n=1 (norms) for M_r (solid top curve) and M_{Lr} (dashed topm curve) up to peak of the crash. Lower two curves show n=1.
- b) Harmonic number N for which at least ½ the total amplitude (in norm) lies in harmonics n≥N. Full MHD M_r (blue), M_L (red). n≤24 total.
- Higher order (∂/∂φ)~*in* terms in M_r become significant as harmonic number rises ((∂ψ/∂φ)` ~nm~n^2>R/r) and bootstraps the generation of yet higher harmonics

Implications: Linear perturbation theory

- Both nonlinear tangle and linear perturbation are "sufficiently small amplitude" approximations, but lead to fundamentally different electromagnetic results
- Linear perturbation theory implicitly assumes that the perturbation is bounded by a flux tube (pert$\rightarrow$0)
- Magnetic field perturbation: Field line equations $dR/B_R=dZ/B_Z=Rd\varphi/B_\varphi$ yields all powers of $e^{in\varphi}$:
  $dR/Rd\varphi = B_R/B_\varphi=(B_{Ro}+\delta B_R e^{in\varphi})(B_{\varphi o}+\delta B_\varphi e^{in\varphi})^{-1}$
  - Linear perturbation is "infinitesimal" amplitude and drops higher powers; linear magnetic islands have zero width! [F. Waelbrock, Nuclear Fusion (2009)]
  - Nonlinear tangle sees an island of the width of the perturbation amplitude

Implications: Gyrokinetic particle models

- Gyrokinetic models approximate the Larmor orbit effects of charged particle motion in a strong **B** field
  - Larmor frequency is fast, so average the magnetic effects on the $\perp$B motion at (4) points around a field line at the Larmor radius ρ
- Beautiful theory extends to all orders in ρ/L in an axisymmetric torus with good nested flux surfaces [Brizzard and Hahm]
- In stochastic field, with island chains, tangle, etc, only the lowest order linear approximation appears to hold
  - Field "parallel" and $\perp$ directions change drastically over a Larmor radius
  - Mostly OK: most simulations are first order
- Related: is it possible to incorporate full electromagnetic effects when magnetic vector potential is expressed as $A_{\|}$, $\mathbf{A}_{\perp}$?

Summary

- Toroidal magnetic fields are Hamiltonian systems
  - Nonlinear small perturbations lead to magnetic near-Hamiltonian stochasticity (magnetic tangle, KAM islands)
- MHD plasma instabilities can provide the required seed perturbation $\perp\mathbf{B}$
- MHD nonlinear mode coupling can also generate its own stochasticity
  - Important instabilities, including ELMs, mix both effects
- Effects are nonlinear --- studies are just beginning
  - Require numerical simulation, tools to analyze stochasticity
  - Presence of magnetic stochasticity has implications for GK particle models, turbulent transport, etc
- Related effects exist in non-toroidal plasmas