ROMA

RANK-ORDERED MULTIFRACTAL ANALYSES

OF INTERMITTENCY IN

SPACE PLASMAS AND THE COSMIC WEB

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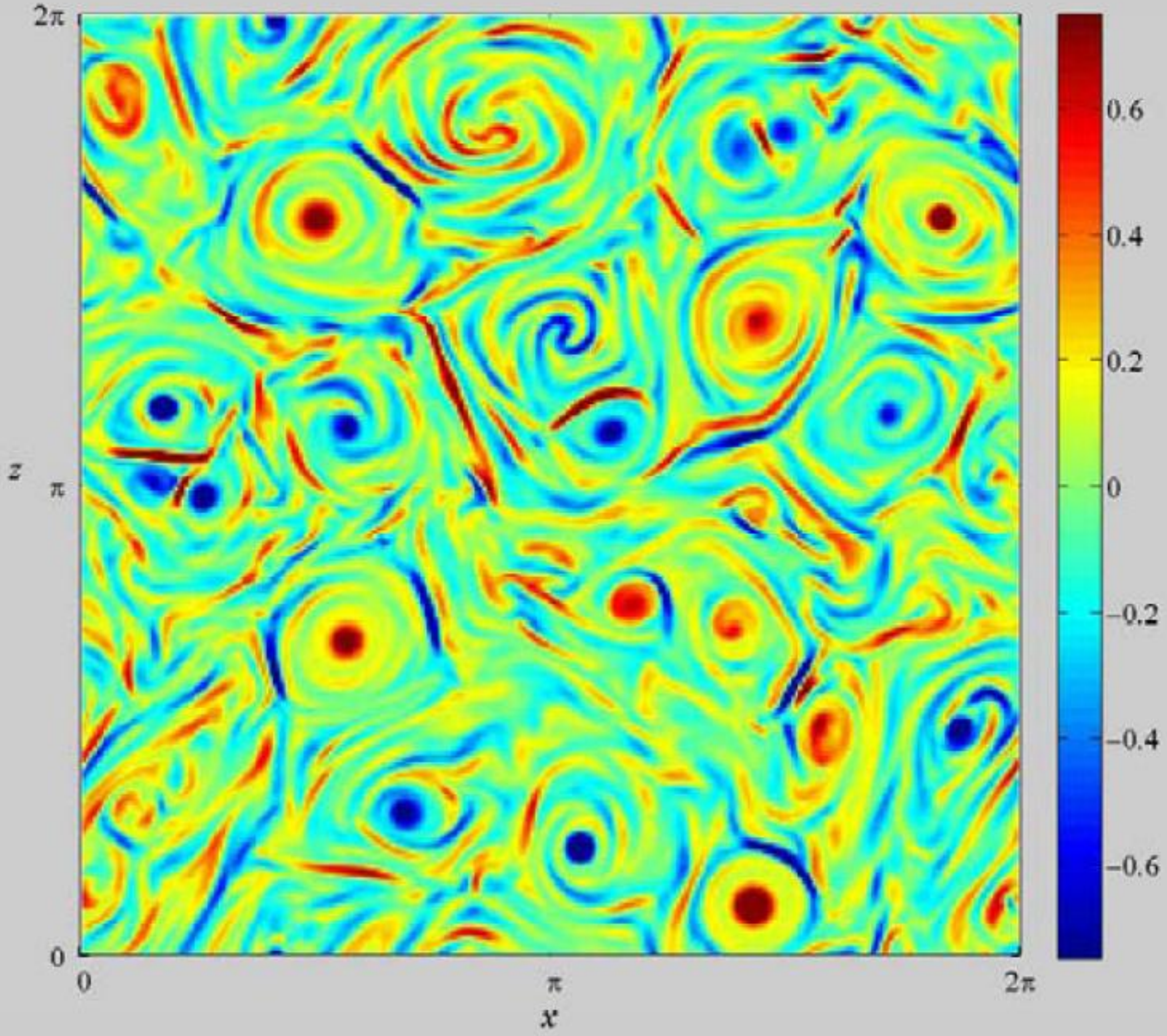
Catholic University of America
NASA Goddard Space Flight Center, Greenbelt
John Podesta
Space Science Institute

-- The Founding of ROMA --

A Local Invariance Theory
of
Dynamic Complexity

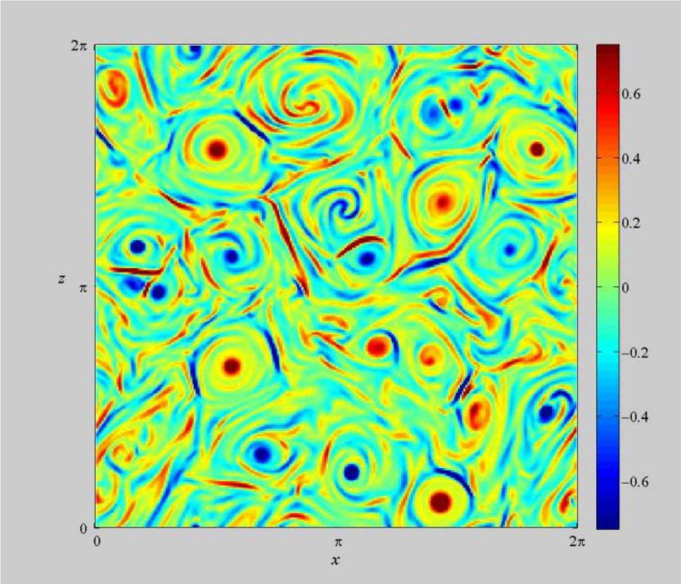


2π
z
π
0
0
π
2π
x
0.6
0.4
0.2
0
−0.2
−0.4
−0.6

FRACTAL

and

INVARIANCE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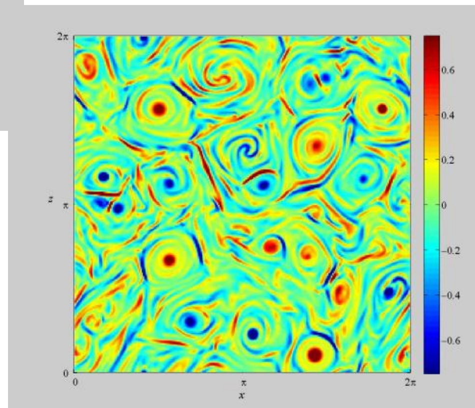
z
π
0
0
π
2π
x
0.6
0.4
0.2
0
−0.2
−0.4
−0.6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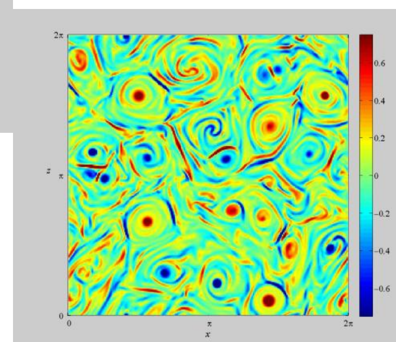
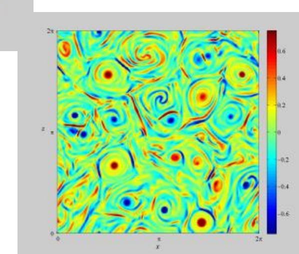
z
π
0
0
π
2π
x
0.6
0.4
0.2
0
−0.2
−0.4
−0.6


2π
z
π
0
0
π
2π
x
0.6
0.4
0.2
0
−0.2
−0.4
−0.6



PARAMETER SET: $\{P\}$

DRG TRANSFORMATION:

$$d\{P\}/dl = \mathcal{R}\{P\}$$

Procedure:

Search for Invariants

LINEAR TRANSFORMATIONS OF $\{P\}$

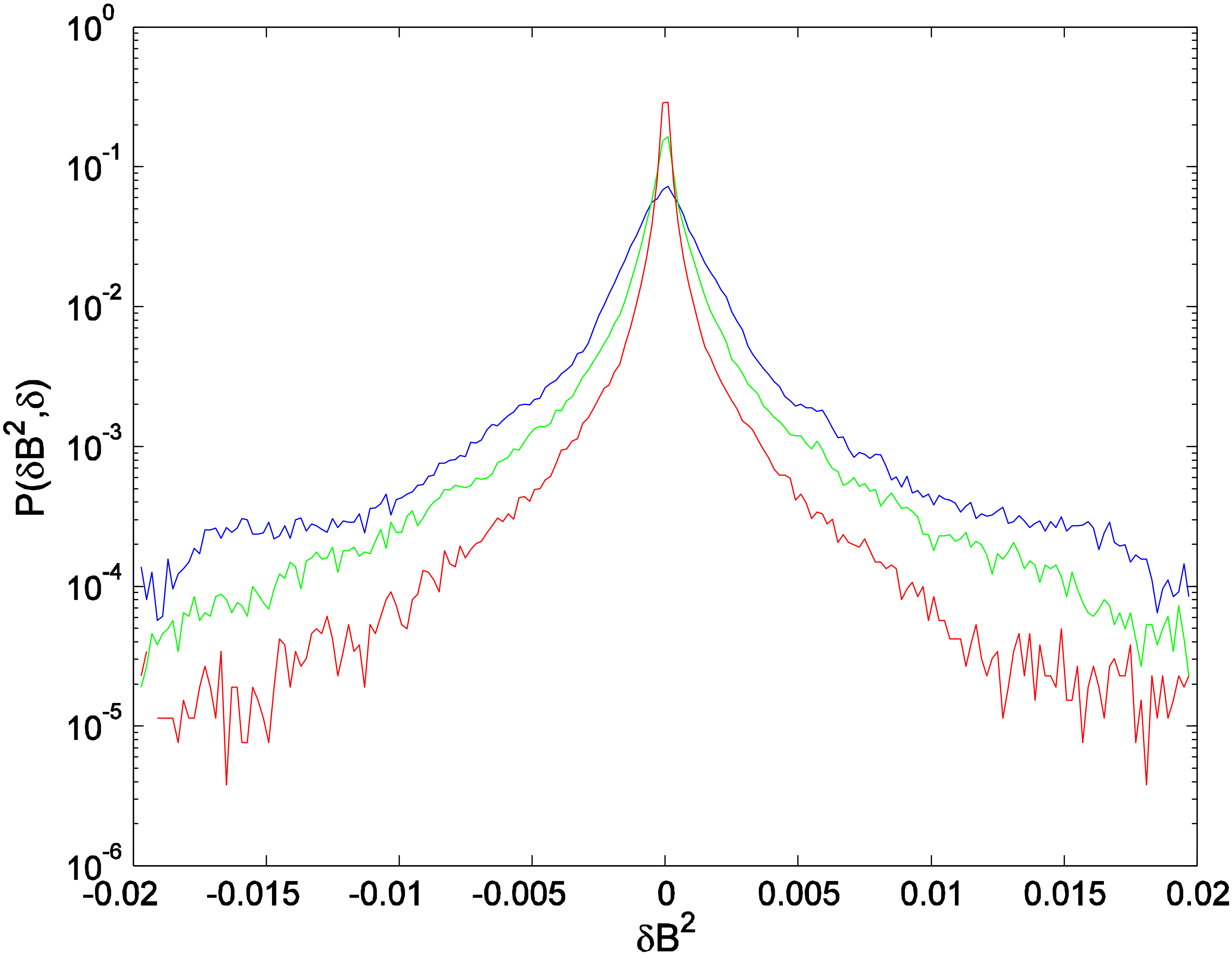
$\rightarrow$ POWER LAW INVARIANTS.

****

<u>*EXAMPLE:*</u>

$B \rightarrow e^{a_B} B, \quad \Delta \rightarrow e^{a_\Delta} \Delta$ *LEADS TO*

$B / \Delta^{a_B / a_\Delta}$ AS AN INVARIANT.



10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
$P(\delta B^2,\delta)$
-0.02
-0.015
-0.01
-0.005
0
0.005
0.01
0.015
0.02
δB^2

$$P(X,\delta)$$

$$P \sim \delta^{a}, \qquad P/\delta^{a} = I$$

$$X \sim \delta^{b}, \qquad X/\delta^{b} = \mathrm{Y}$$

[Chang et al., 1973)]

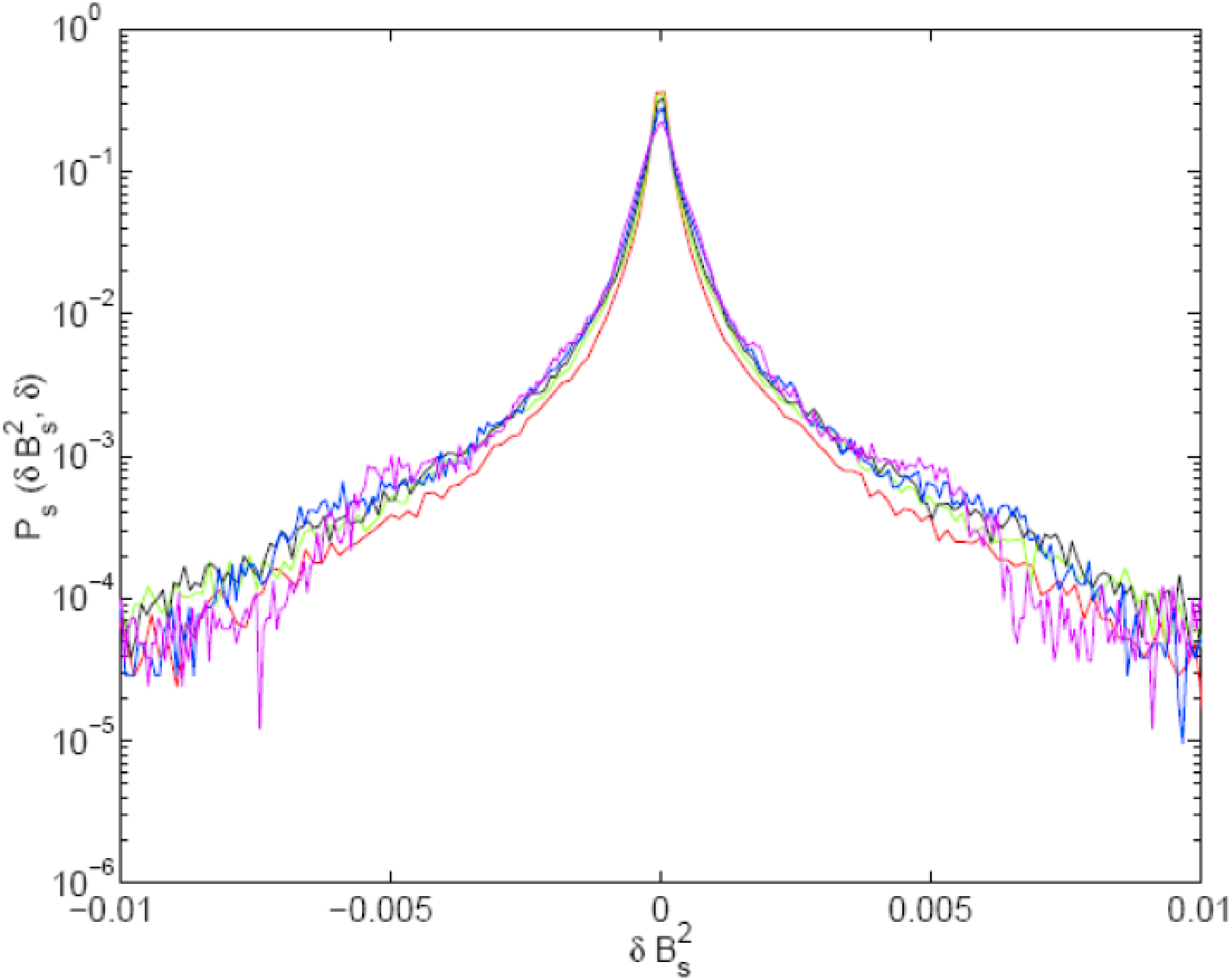
INVARIANCE OF FUNCTIONAL FORM

$P(X,\delta)$ LEADS TO:

$\rightarrow$

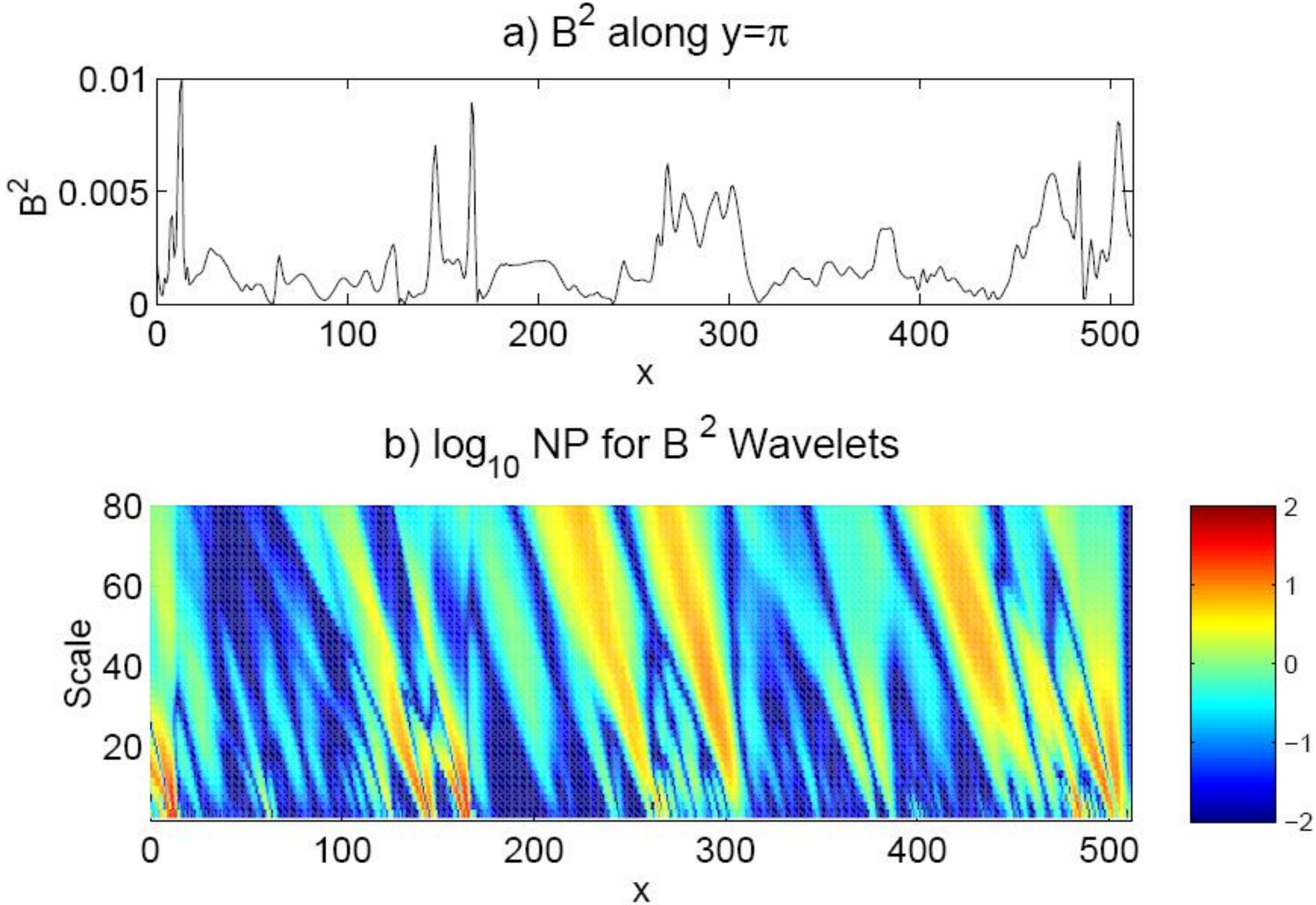
$$I = \mathsf{F}(\mathrm{Y})$$

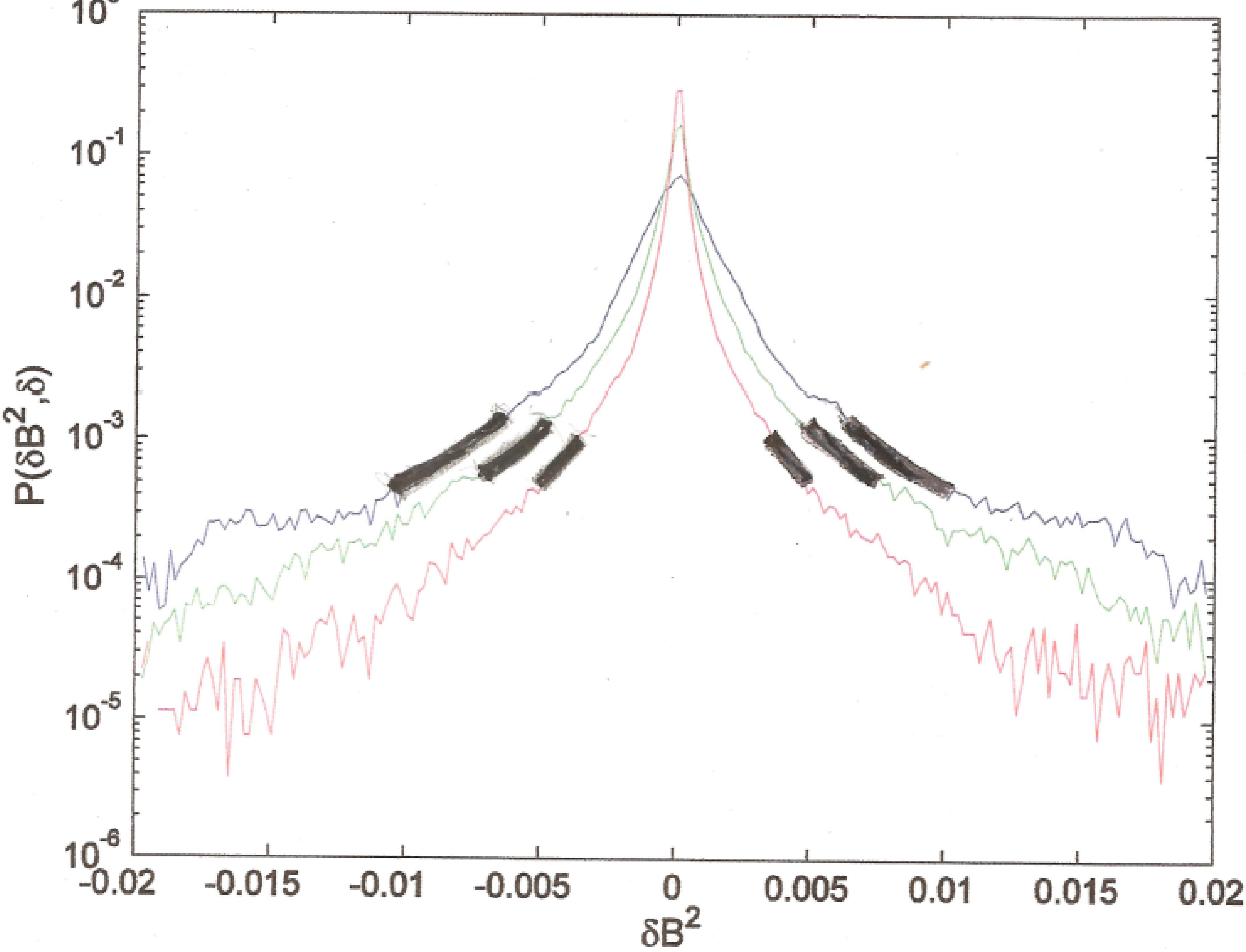
$$P/\delta^{-s} = \mathsf{F}(X/\delta^{s})$$



CHANG ET AL., 2004, HNAT ET AL., 2002, Solar Wind

HAAR WAVELETS





10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
$P(\delta B^2,\delta)$
-0.02
-0.015
-0.01
-0.005
0
0.005
0.01
0.015
0.02
δB^2

$$P\delta^{s} = \mathsf{F}(X/\delta^{s})$$

$$\rightarrow \qquad M(q) \sim \delta^{qs}$$

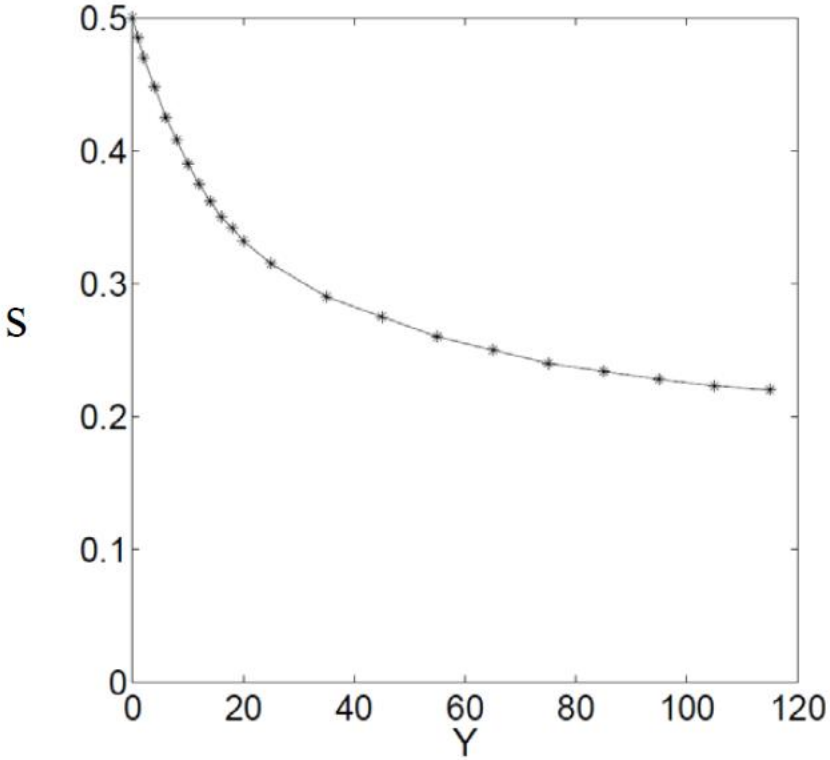
S: *Hurst Exponent*

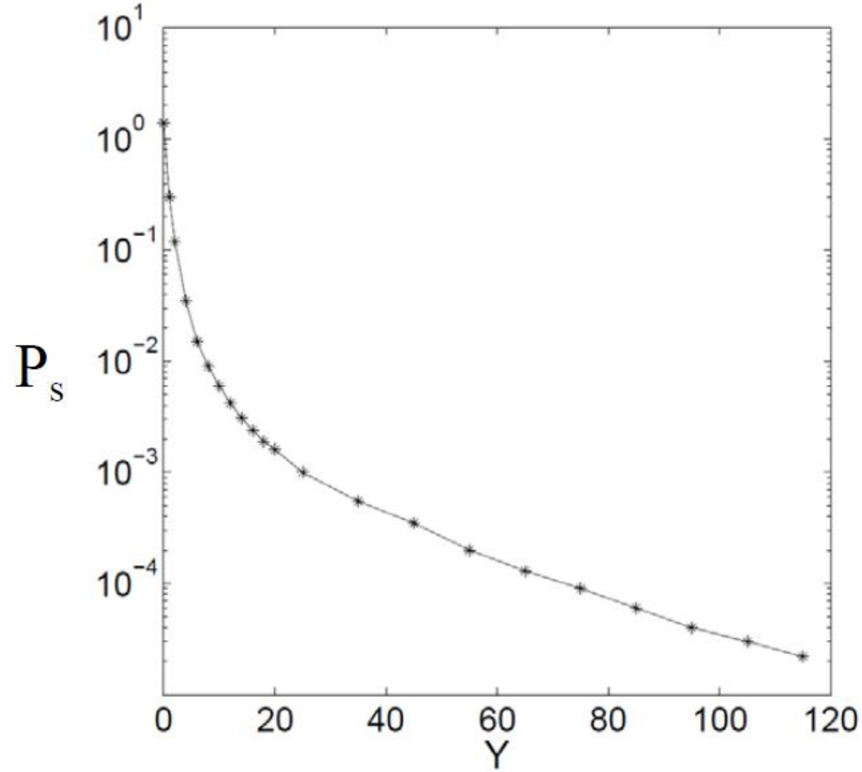
Multifractals:

s(Y):Implicit Spectrum,

$$\mathrm{Y} \equiv X/\delta^{s}$$

$$P = P(\delta X, \delta) \rightarrow \begin{matrix} s = s(Y) \\ P_s = P_s(Y) \end{matrix}$$





Auroral Electric Field Fluctuations (SIERRA):

Sunny Tam (PSSC, NCKU, Taiwan)

Cusp and Magnetosheath Regions (Cluster data):

Hervé Lamy and Marius Echim (Belgium ISA)

Solar Wind (Wind):

John Podesta (EOS, UNH)

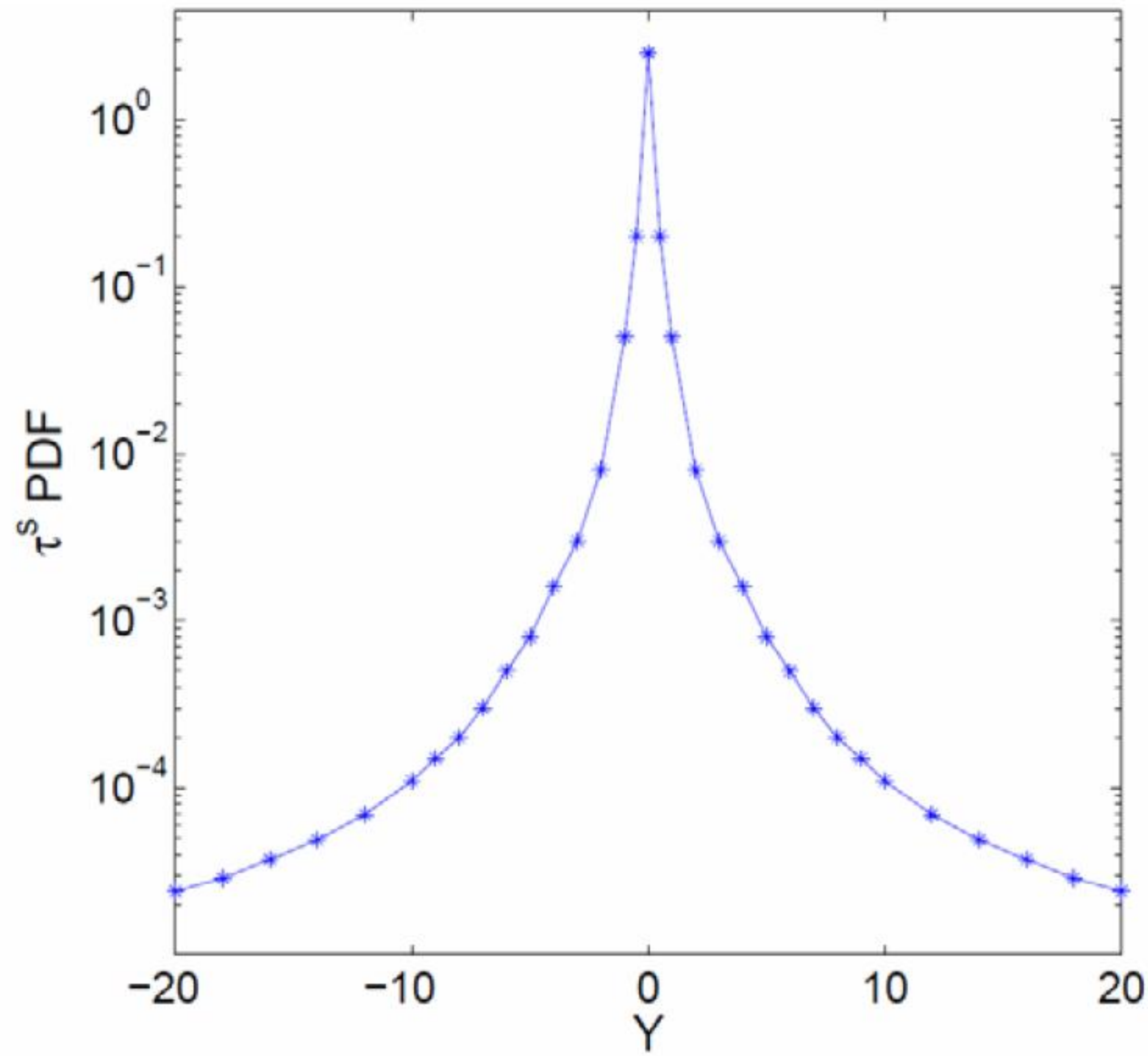
AE Index:

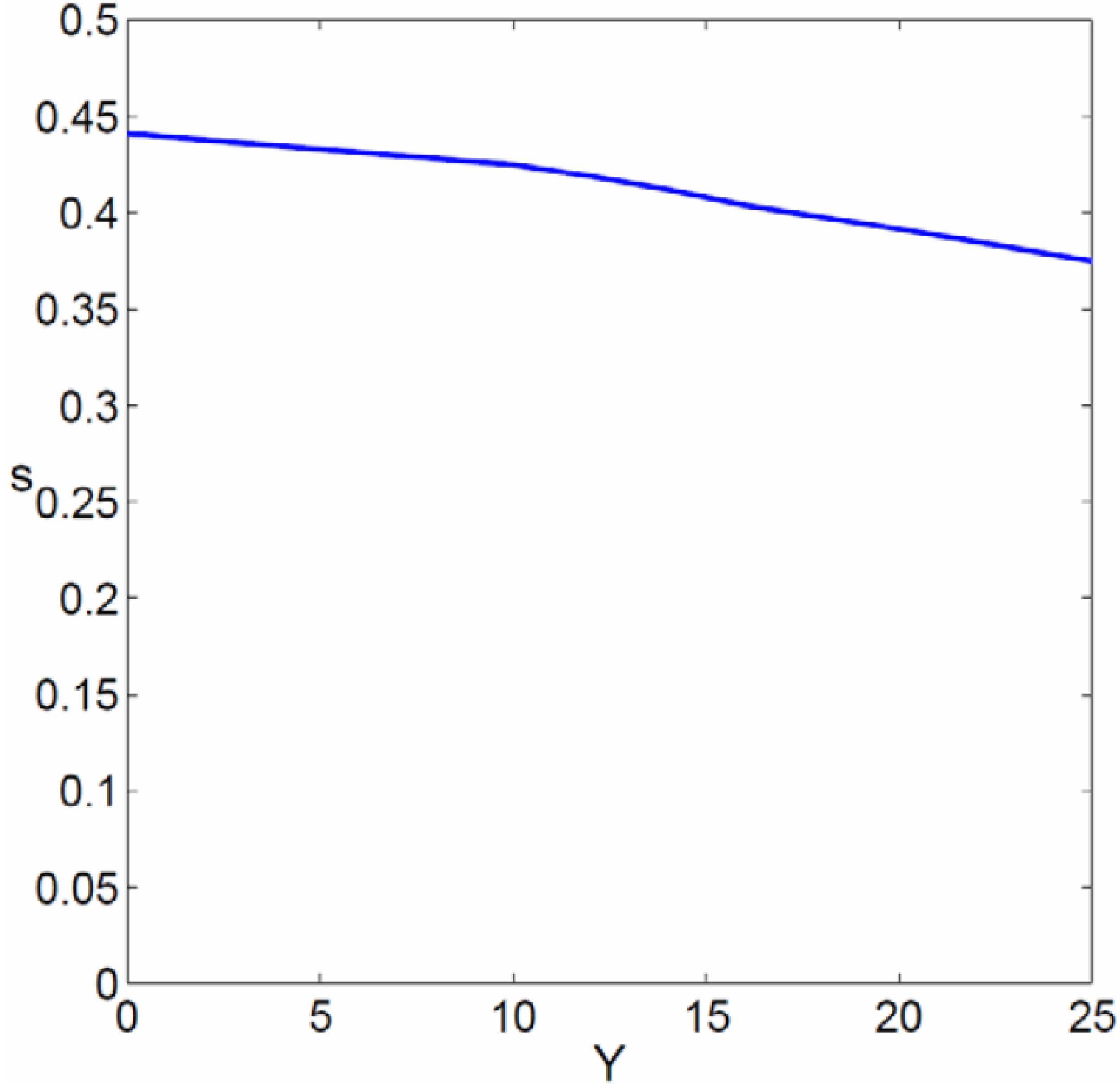
Giuseppe Consolini (IFSI/INAF)

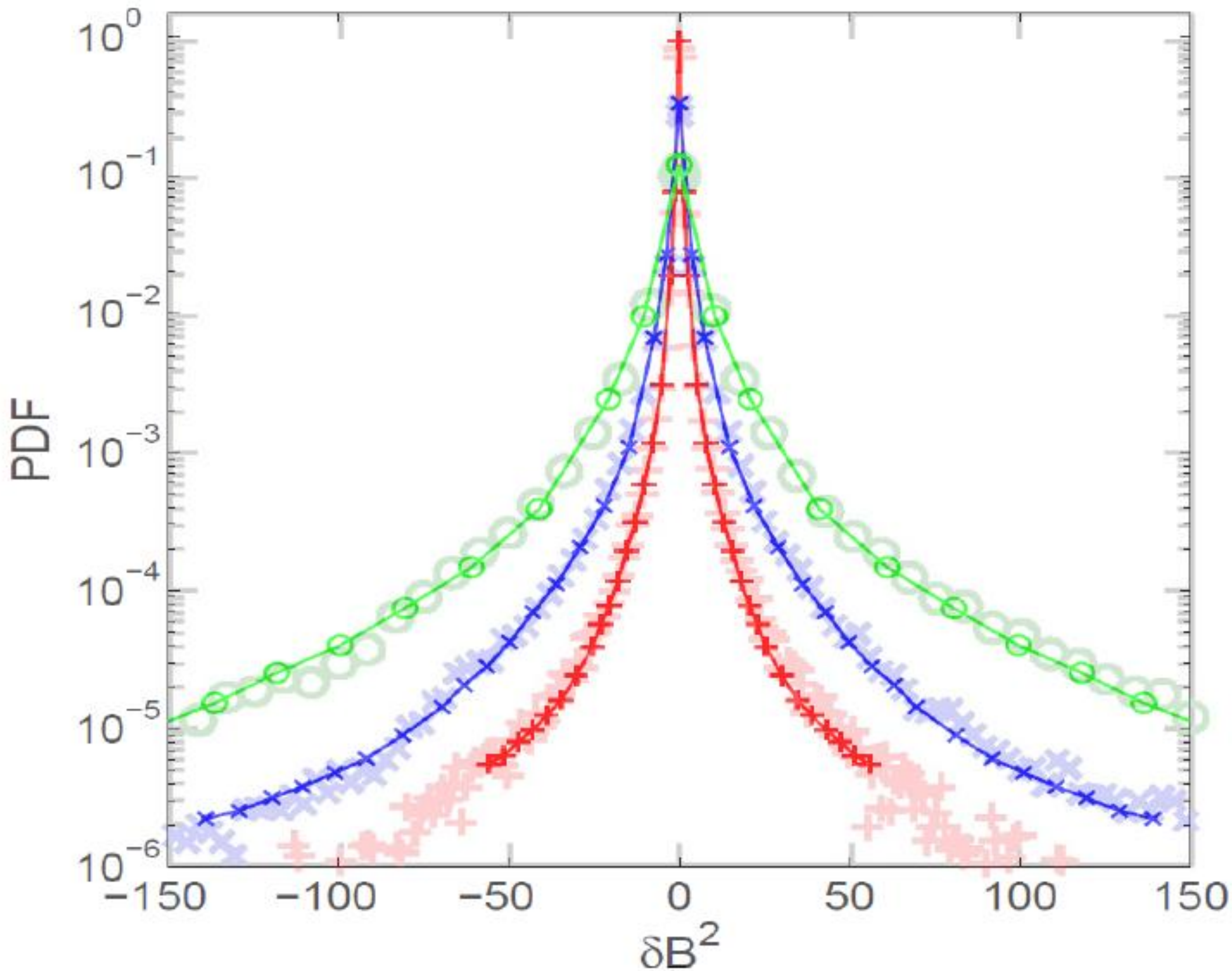
Corona EUV Luminosity

Vadim Uritsky (CUA/GSFC)

SOLAR WIND




0.5
0.45
0.4
0.35
0.3
s
0.25
0.2
0.15
0.1
0.05
0
0
5
10
15
20
25
Y



Computed PDFs from the scaling relations are shown in the front; data are shown in the back.

Green (o): τ=1000s; blue (x): 96s; red(+): 9s.

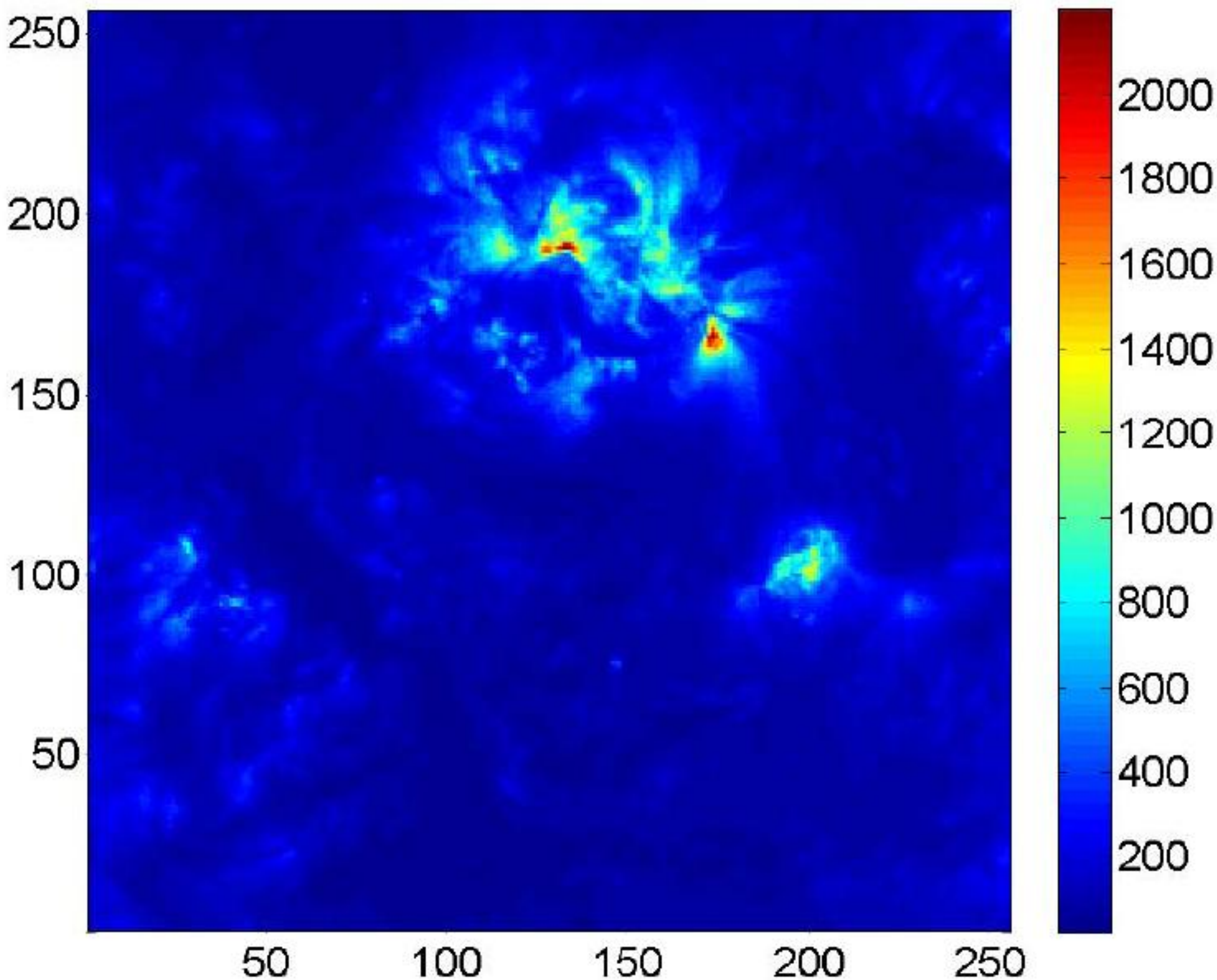
SOHO extreme ultraviolet imaging telescope (EIT) data:

We have studied a set of full-disk digital images of the corona taken by the extreme ultraviolet imaging telescope (EIT) on board the SOHO spacecraft* in the 195Å wavelength band corresponding to the Fe XII emission at peak coronal temperatures of 1.6×10^6 K. The data included one observation period: 06/29/2001–07/28/2001 (3240 images, solar max, average sunspot number 64.0) with a typical time resolution of 13.3 min. To reduce optical distortions, we studied only central portion of the Sun disk with the linear dimensions 1040 x 1040 Mm (256 x 256 pixels with 5.6 arcsec resolution).

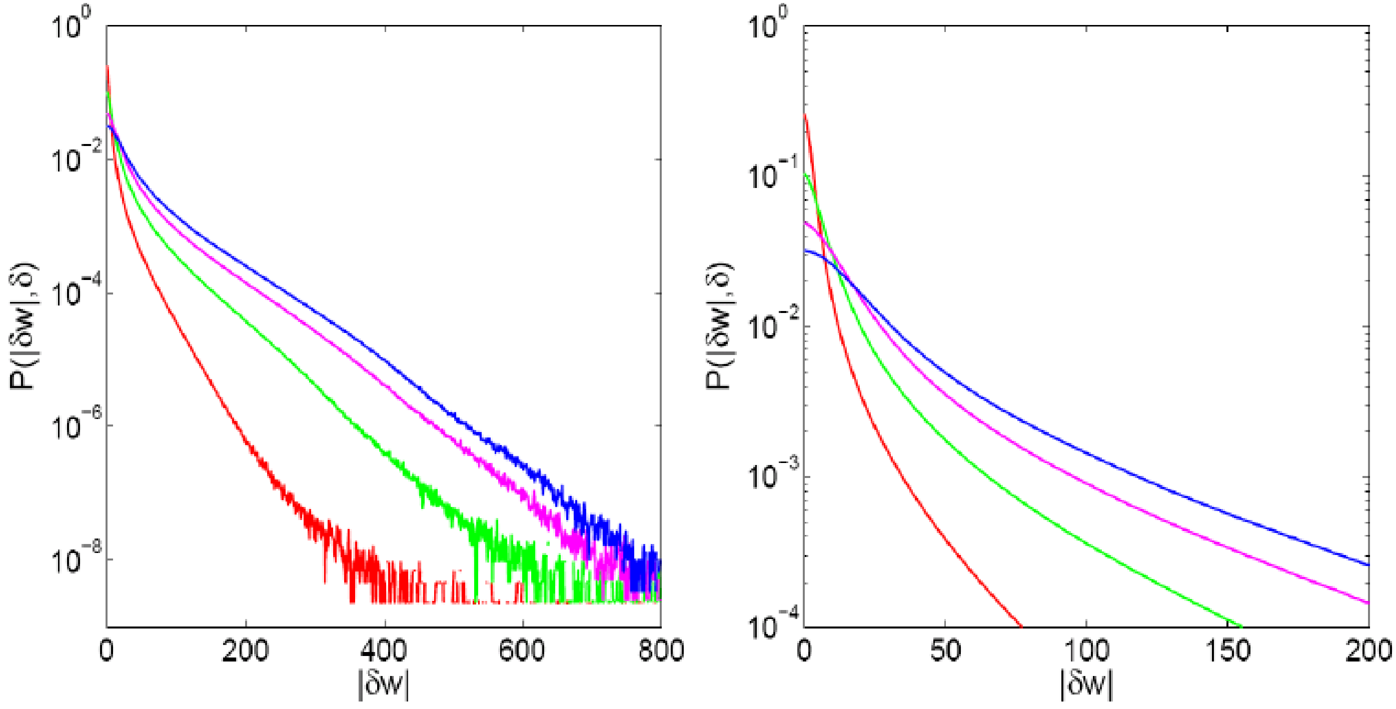
The EIT luminosity $w(t; \boldsymbol{r})$ was analyzed as a position $\boldsymbol{r}$ on the image plane. The dynamics of $w(t; \boldsymbol{r})$ captures the redistribution of radiative flux in consecutive EIT frames due to a variety of coronal features such as loops and holes, mass ejections, etc. For the purpose of this study, we treated $w(t; \boldsymbol{r})$ as a local measure of coronal activity and did not filter the data in an attempt to distinguish between different types of coronal events.

* J.-P. Delaboudiniere *et al.*, Sol. Phys. **162**, 291 (1995).

Corona EUV Emissions – SOHO EUV emissions

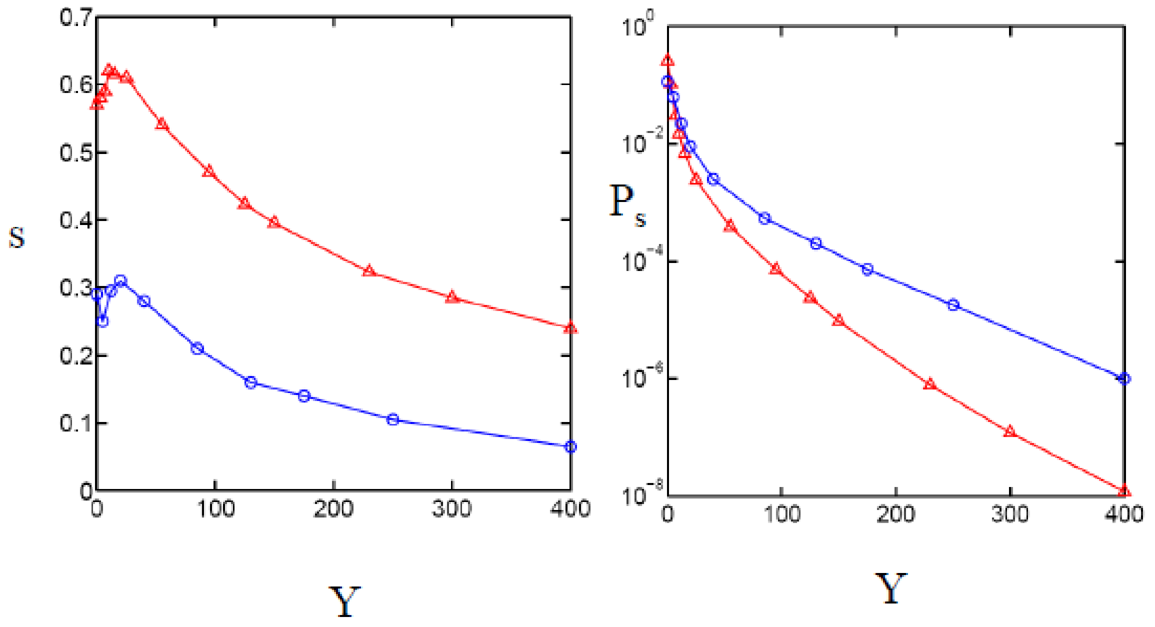


ROMA analysis of probability distributions for the SOHO EIT data I



PDF as a function of the scale: δ=1 (red), 5 (green), 20 (magenta), and 80 (blue). The right panel is an enlarged view.

ROMA analysis of probability distributions for the SOHO EIT data II

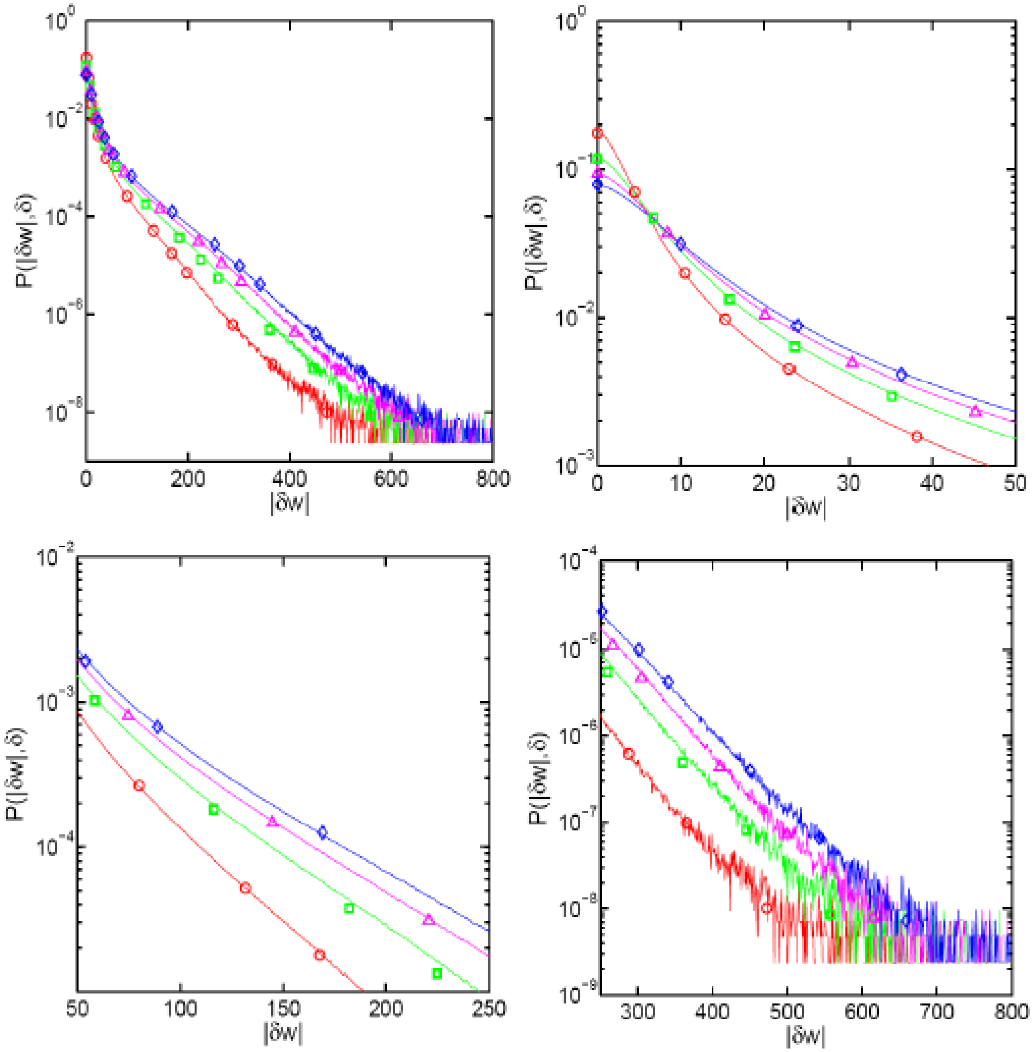


Red triangles: δ from 2 to 8; blue circles: δ from 20 to 80.

In this study, the SOHO EIT data set is shown to include two multifractal rank-ordered regimes, one for δ from 2 to 8 and the other for δ from 20 to 80.

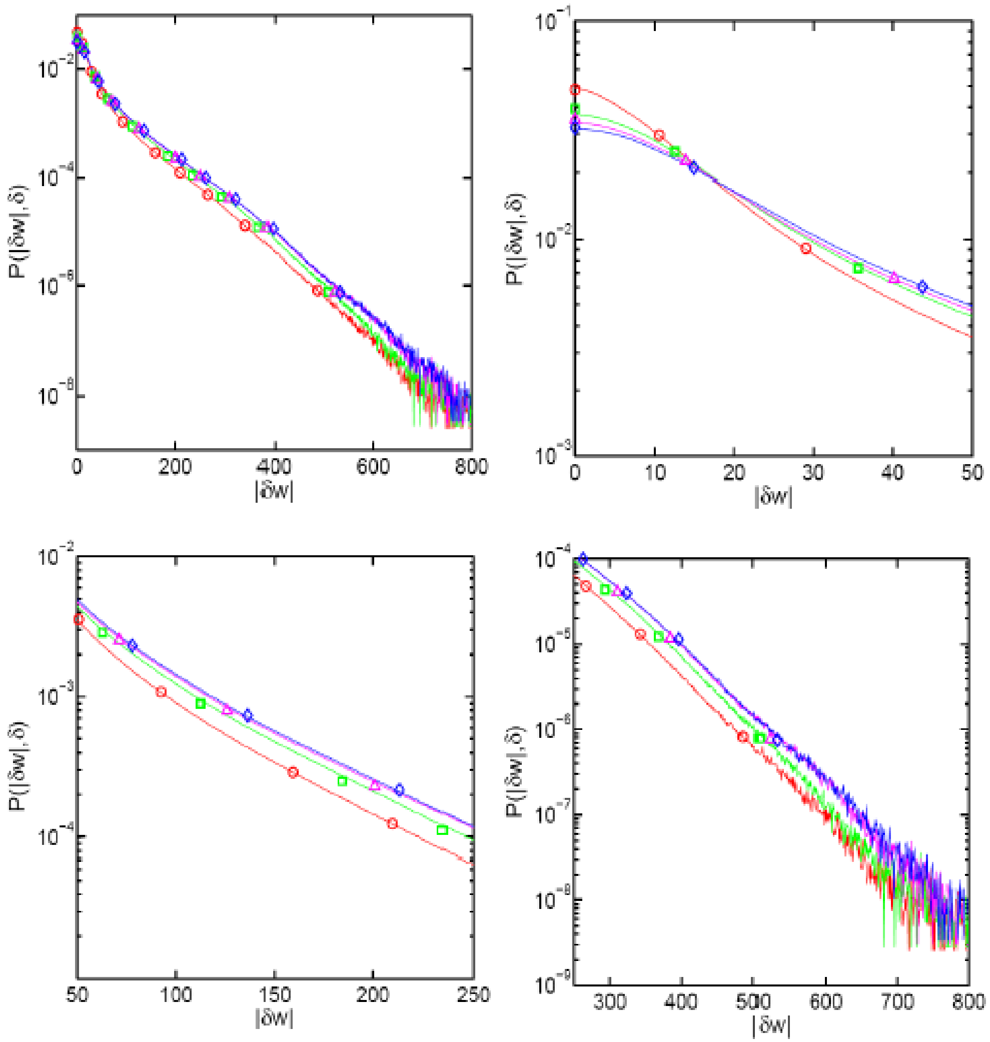
In the next two pages, the two functions $s(Y)$ and $P_s(Y)$ reproduce well the PDFs using the ROMA scaling relations in each respective spatial ranges.

ROMA analysis of probability distributions for the SOHO EIT data III



For $\delta = 2 - 8$; PDF for $\delta=2$ (red), 4 (green), 6 (magenta) and 8 (blue); solid curves from data; markers from the ROMA scaling.

ROMA analysis of probability distributions for the SOHO EIT data IV



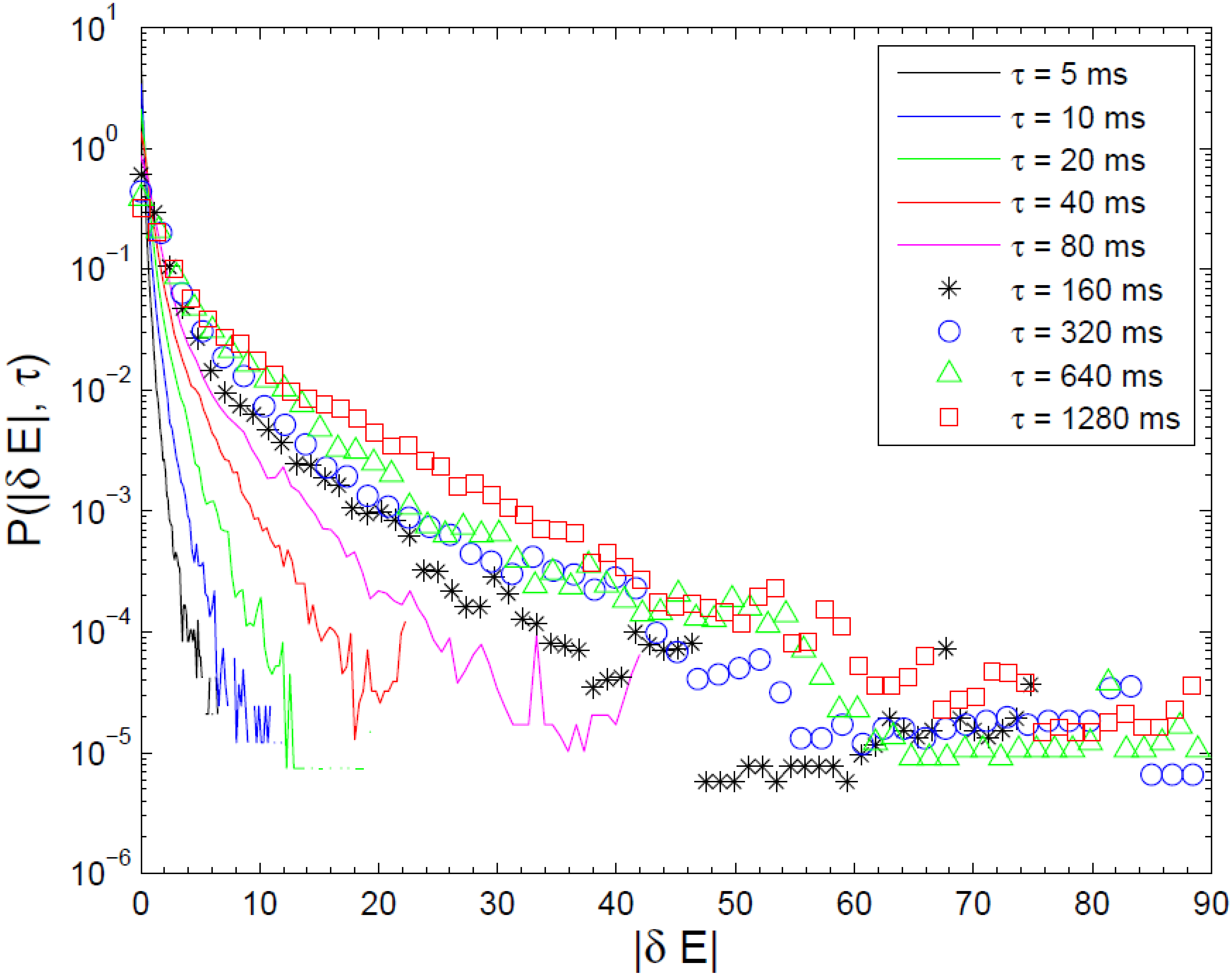
For $\delta = 20 - 80$; PDF for $\delta=20$ (red), 40(green), 60 (magenta) and 80 (blue); solid curves from data; markers from the ROMA scaling.

ROMA Analysis of

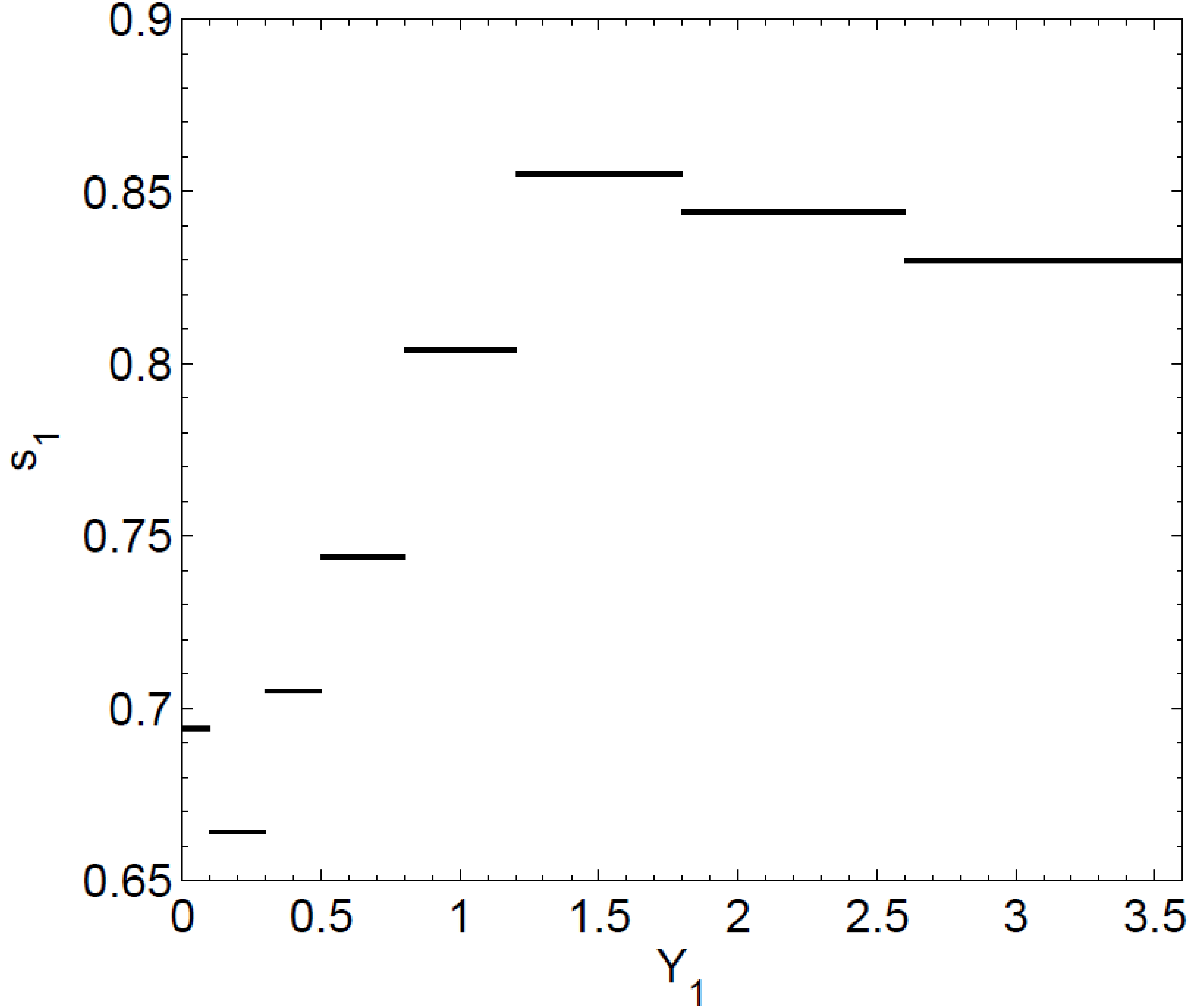
BBELF (Broadband Extremely Low Frequency Fluctuations)

in the Auroral Zone

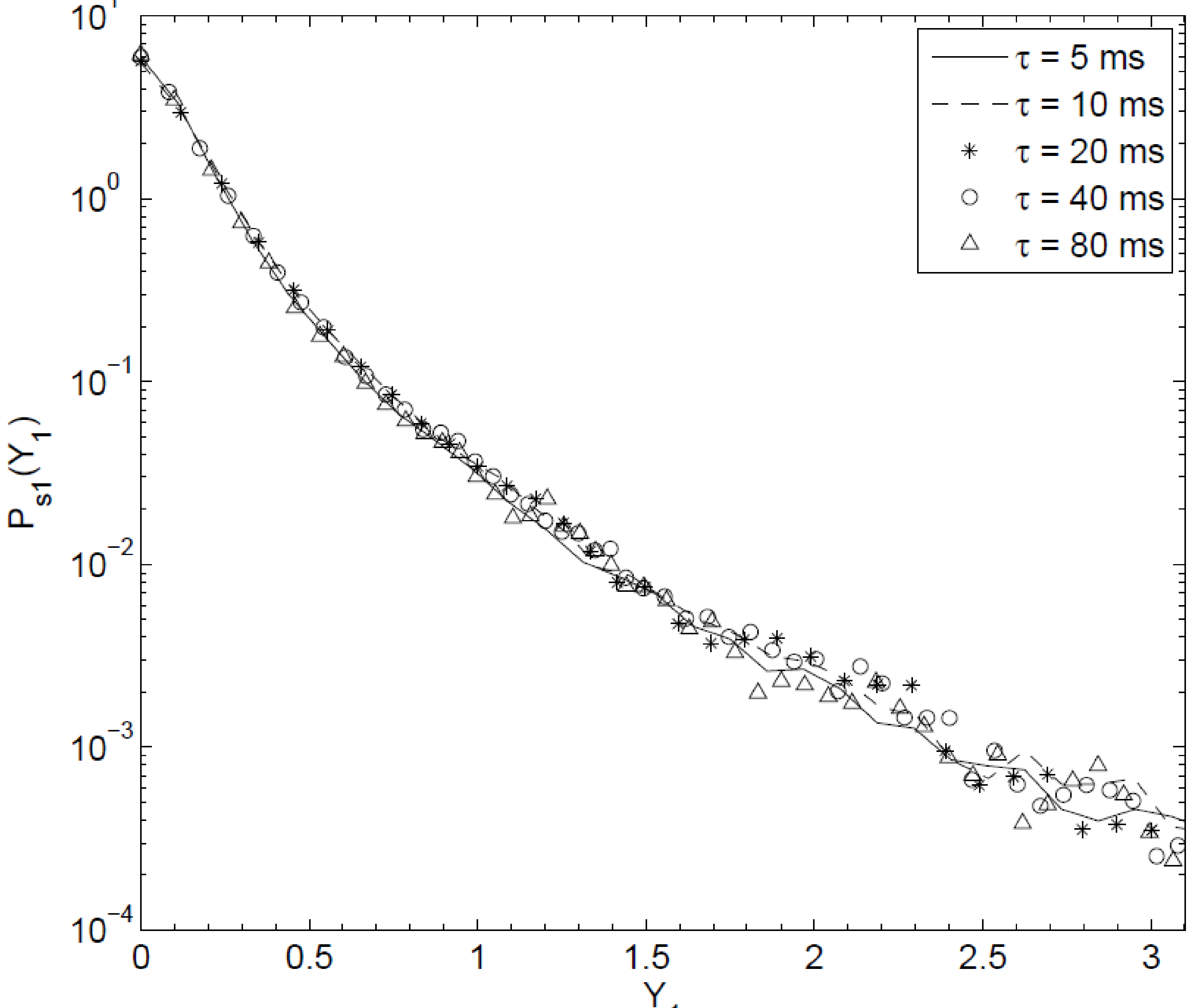
Observations by SIERRA Sounding Rocket (Kintner and Klatt)



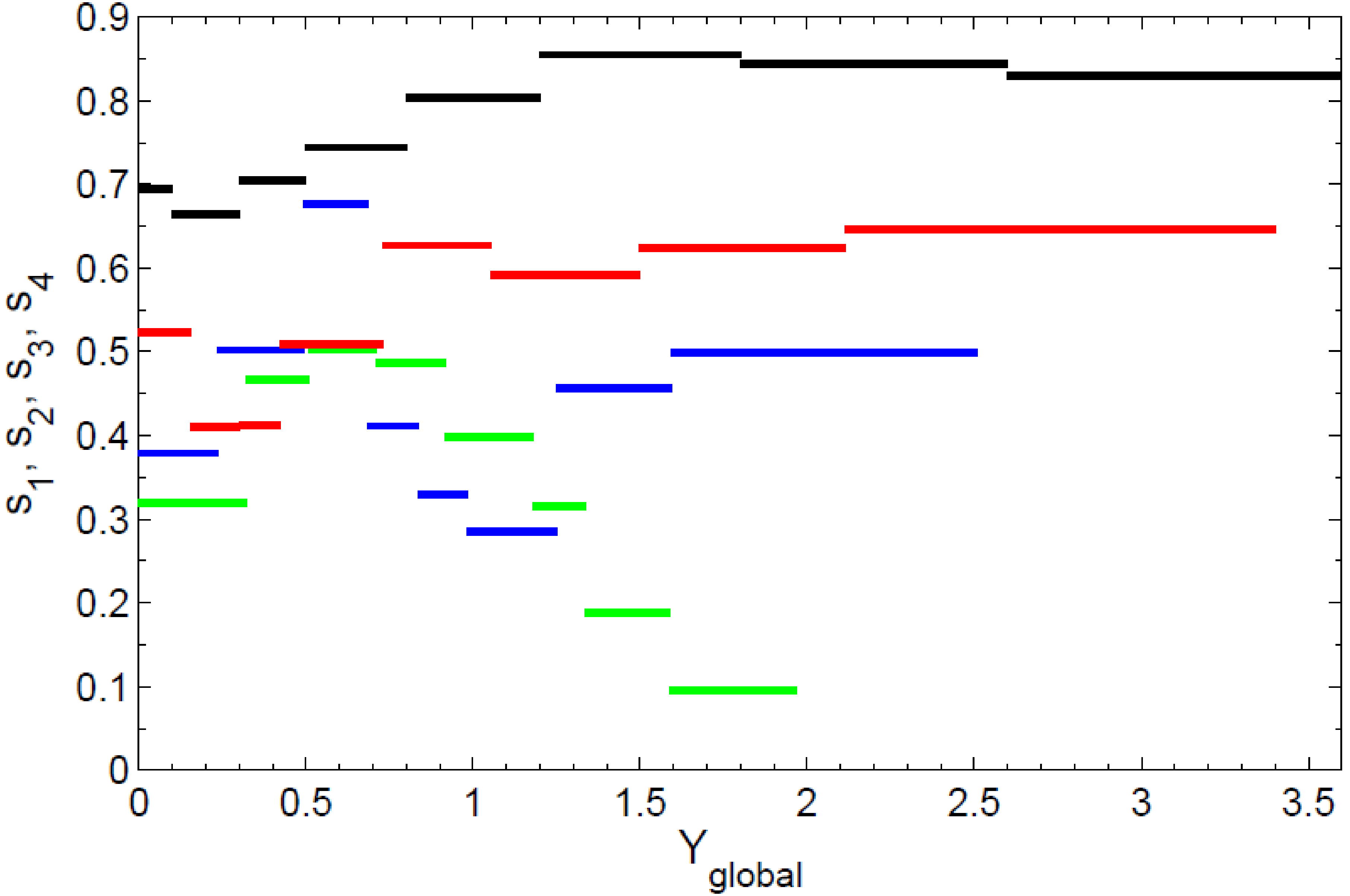
P(|δ E|, τ)
|δ E|
0
10
20
30
40
50
60
70
80
90
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
τ = 5 ms
τ = 10 ms
τ = 20 ms
τ = 40 ms
τ = 80 ms
τ = 160 ms
τ = 320 ms
τ = 640 ms
τ = 1280 ms



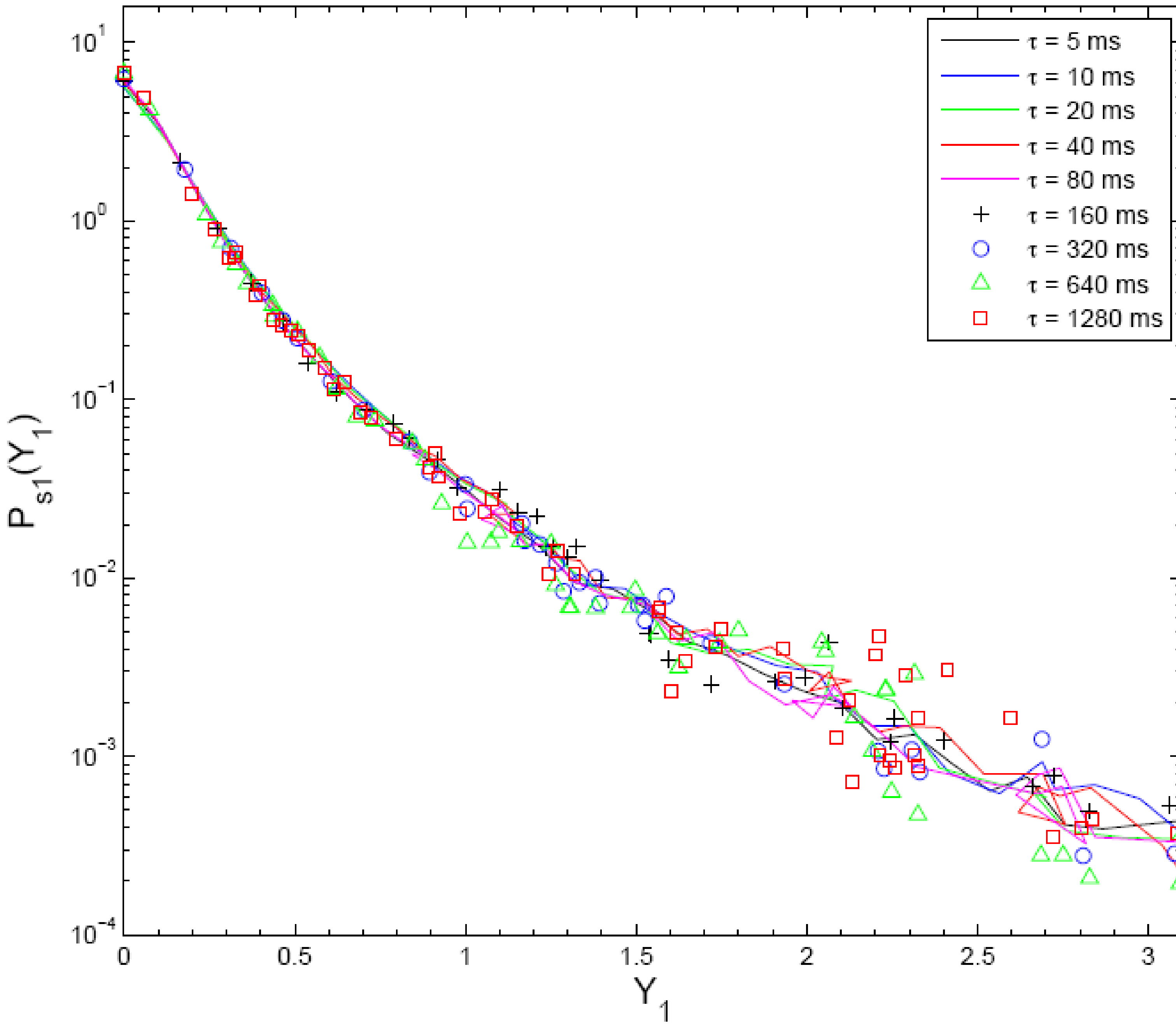
0.9
0.85
0.8
0.75
0.7
0.65
s_1
0
0.5
1
1.5
2
2.5
3
3.5
Y_1



τ = 5 ms
τ = 10 ms
τ = 20 ms
τ = 40 ms
τ = 80 ms
$P_{s1}(Y_1)$
Y_1
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
0
0.5
1
1.5
2
2.5
3



0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
s_1, s_2, s_3, s_4
0
0.5
1
1.5
2
2.5
3
3.5
Y_{global}



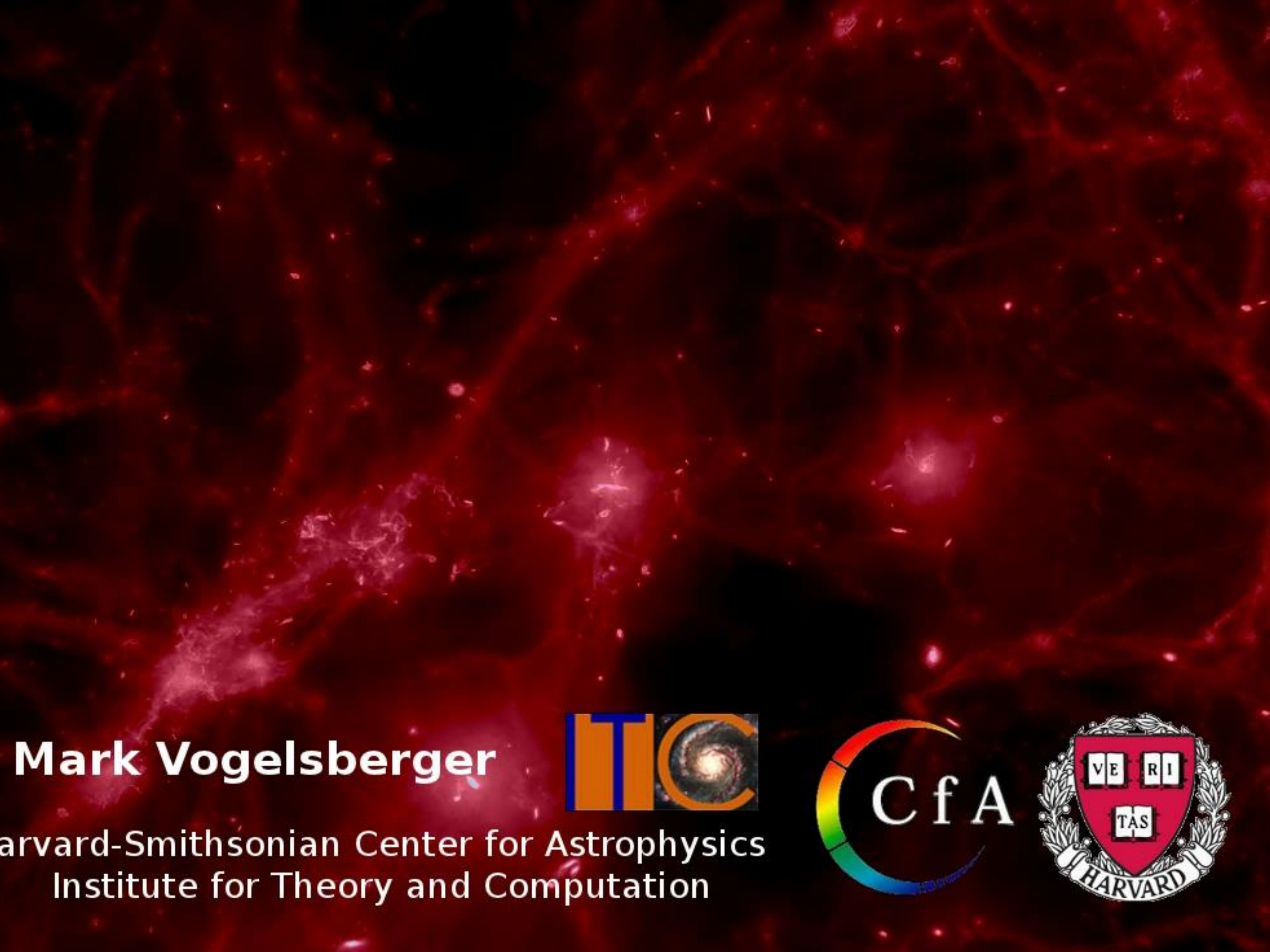
τ = 5 ms
τ = 10 ms
τ = 20 ms
τ = 40 ms
τ = 80 ms
τ = 160 ms
τ = 320 ms
τ = 640 ms
τ = 1280 ms
$P_{s1}(Y_1)$
Y_1
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
0
0.5
1
1.5
2
2.5
3

Complexity Phenomenon of the Cosmic Gas

AREPO Moving Mesh Simulation

Volgersberger et al., 2012

SATOR
AREPO
TENET
OPERA
ROTAS

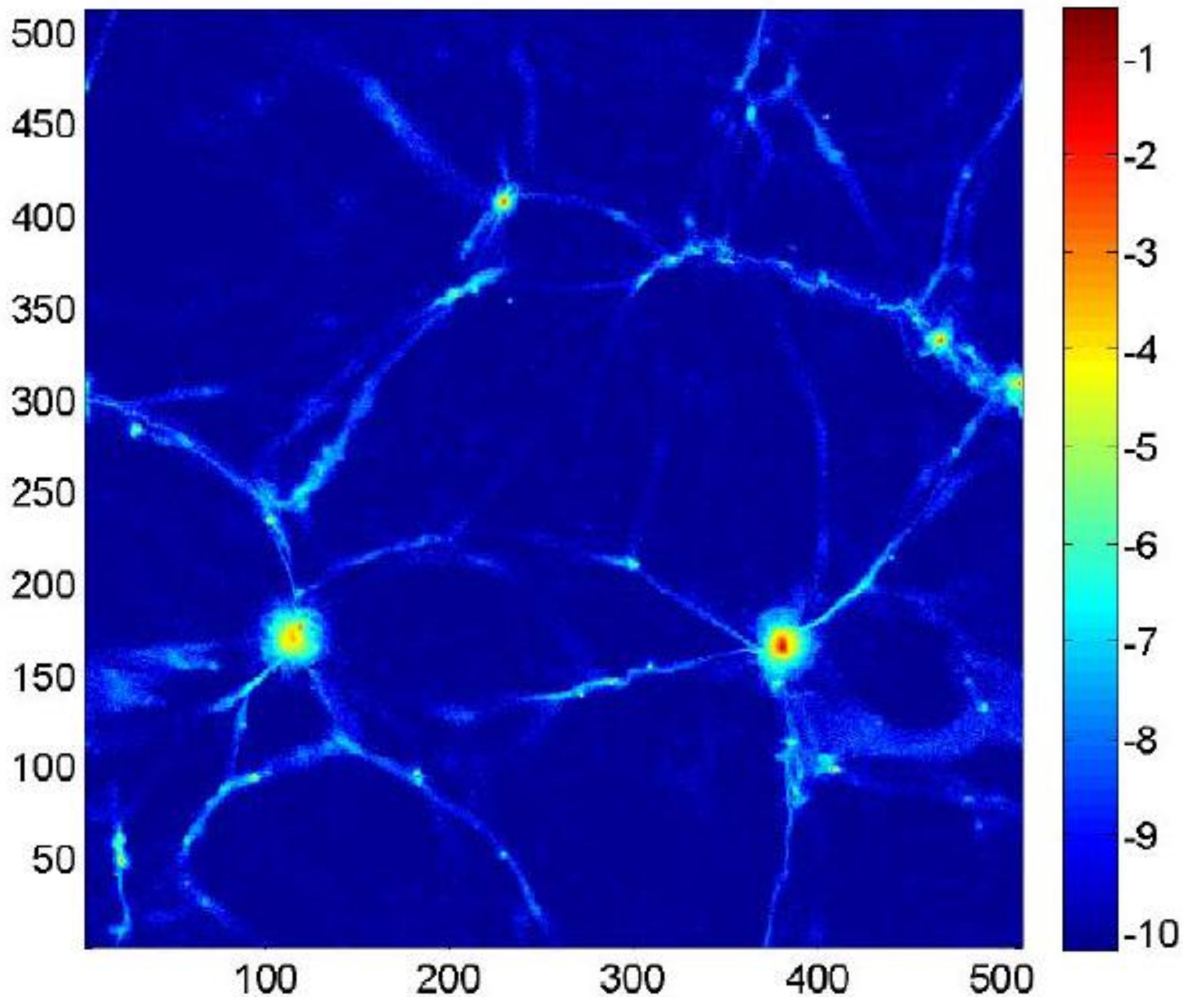


Mark Vogelsberger

arvard-Smithsonian Center for Astrophysics
Institute for Theory and Computation

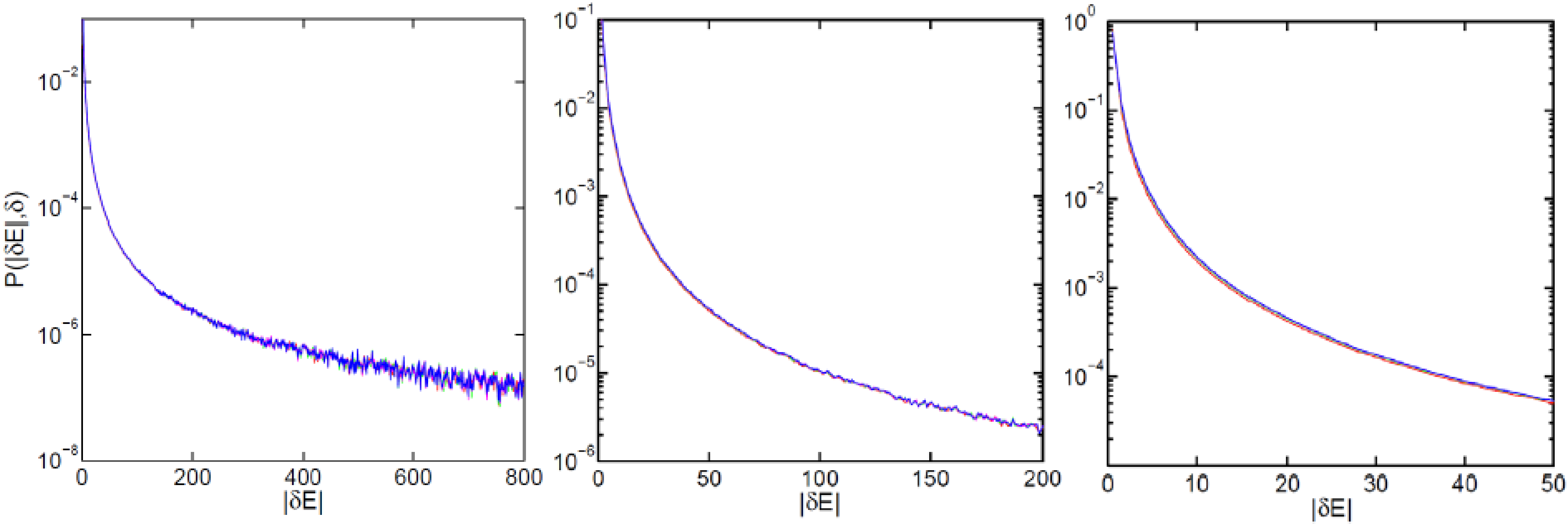
Gas particles in 512^3 cells

Density distributions at $0<x<\Delta$, with $\Delta=20/512$ Mpc/h



$\ln(\rho)$ as function of (y, z);

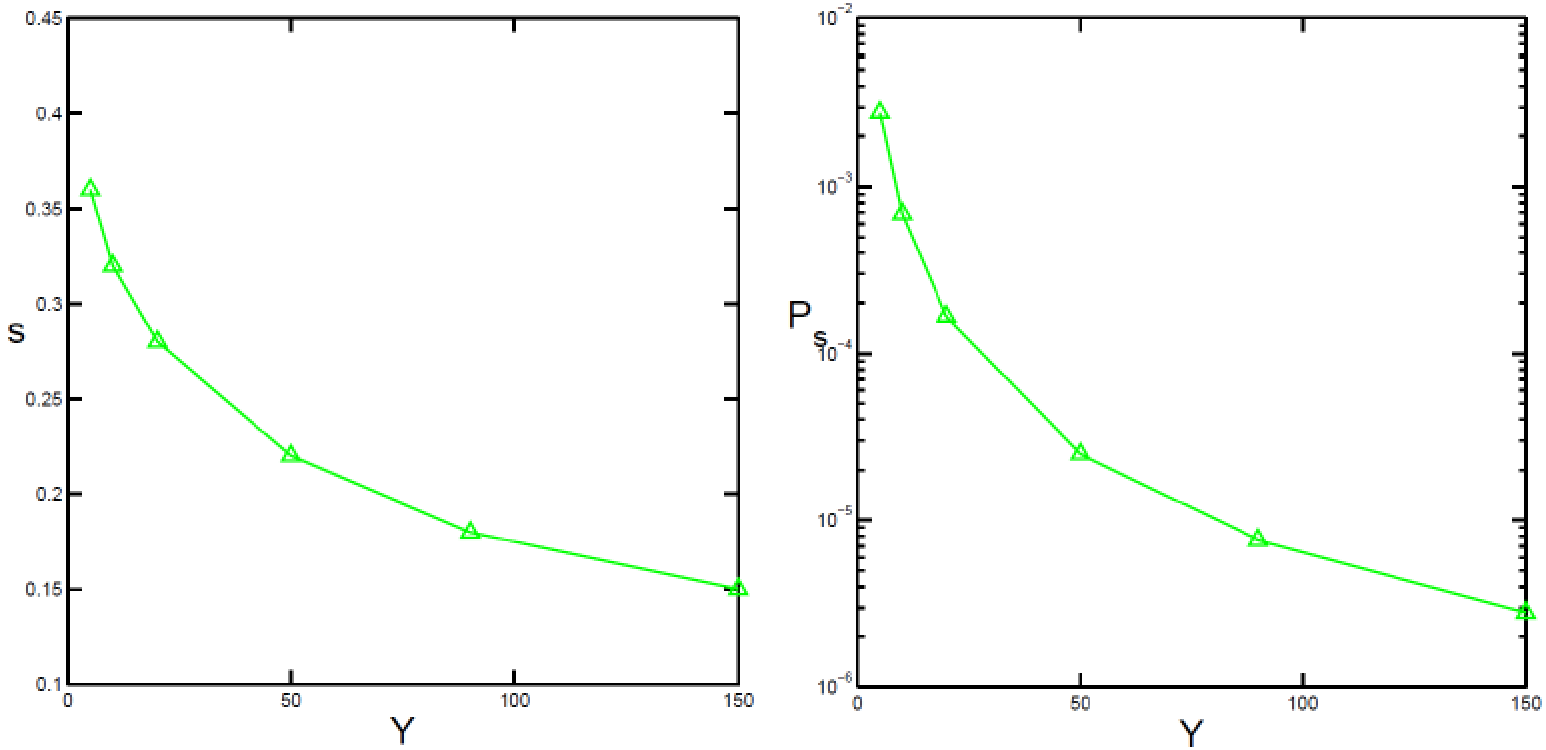
(Ie) PDF of E ($\equiv 1/2\, \rho v^2$) fluctuations at δ=32, 64, 96 and 128Δ, with units=1000/800.



δ=32Δ : red; δ=64Δ : green; δ=96Δ : magenta; δ=128Δ : blue

→ Scale independence

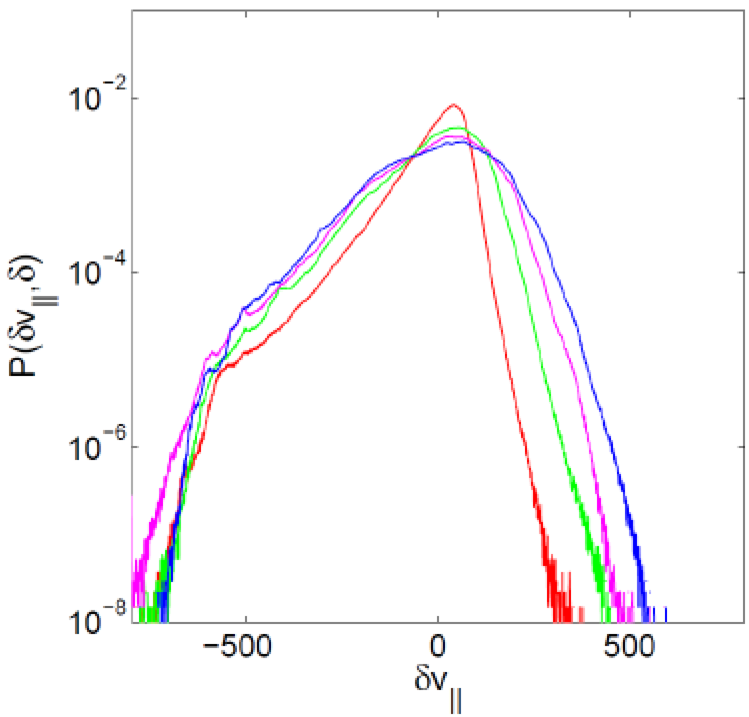
(Id) PDF of E (≡1/2 ρv^2) fluctuations at δ=4, 6, 8 and 12Δ, with units=1000/800.



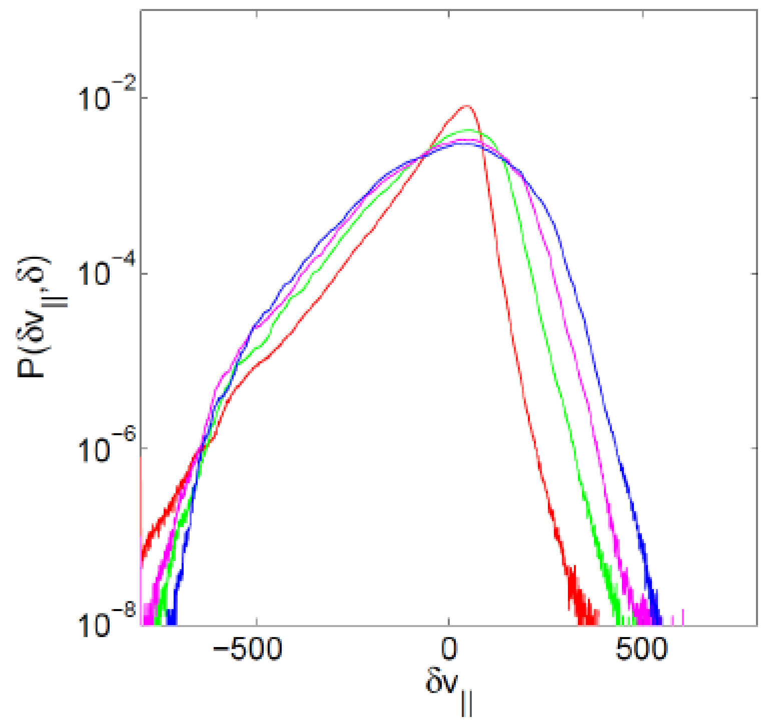
Values of s(Y) and P_s(Y) used in ROMA scalings
in obtaining PDFs shown in the previous slides.
Reasonable values for δ=4 - 12Δ.

(Ib) PDF of $v_{||}$ fluctuations at δ=32, 64, 96 and 128Δ

(c) Based on v_z



(d) Average of (a-c)



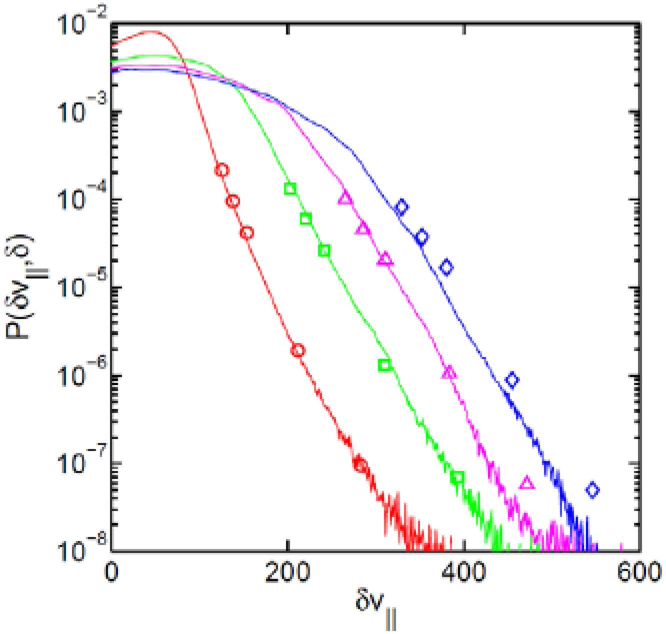
δ=32Δ : red; δ=64Δ : green

δ=96Δ : magenta; δ=128Δ : blue

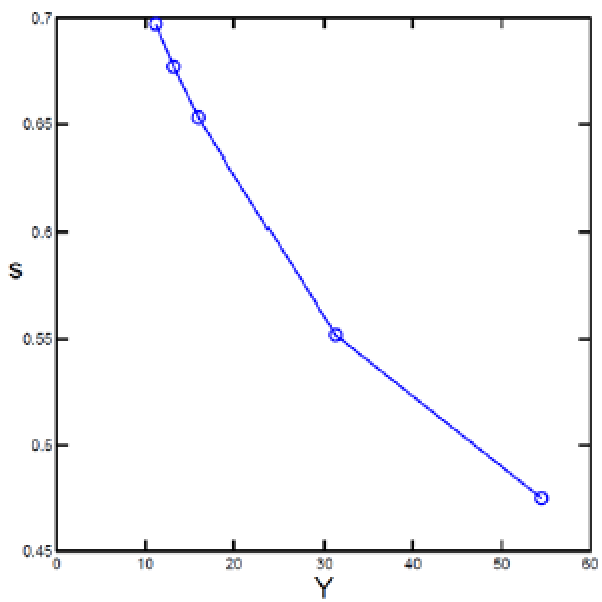
(IIa) PDF of $v_{||}$ fluctuations at δ=32, 64, 96 and 128Δ (the units of $\delta v_{||}$ = 900/800)

(i) $\delta v_{||} > 0$

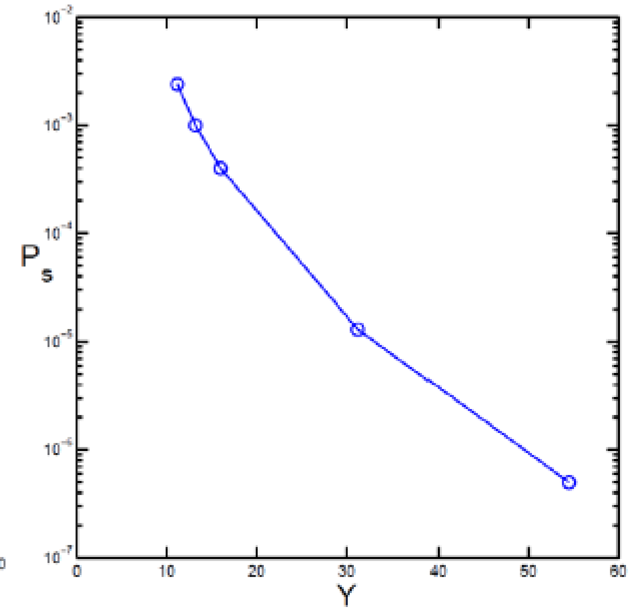
(a) pdf



(b) s(Y)



(c) $P_s(Y)$

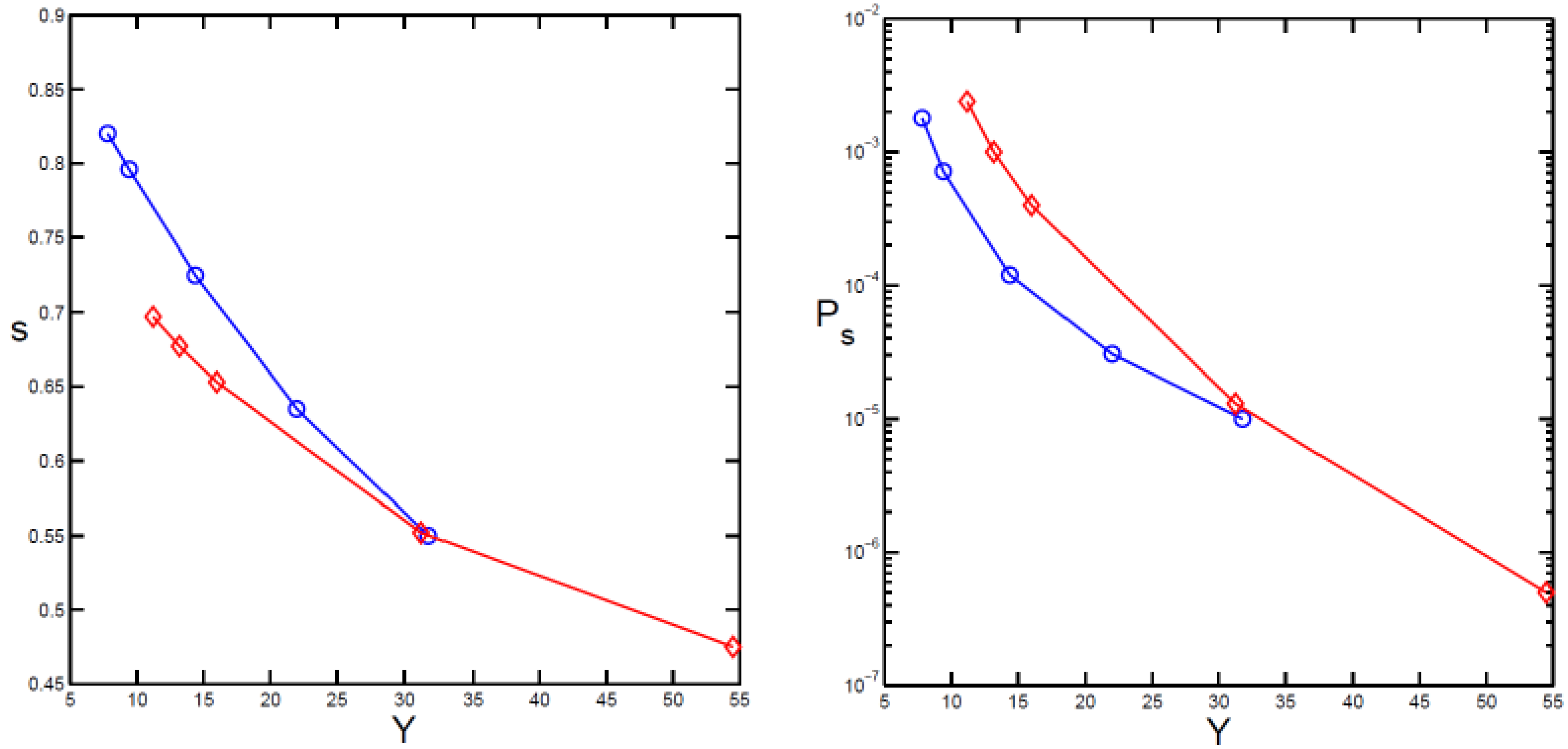


δ=32Δ : red circles; δ=64Δ : green squares

δ=96Δ : magenta triangles; δ=128Δ : blue diamonds

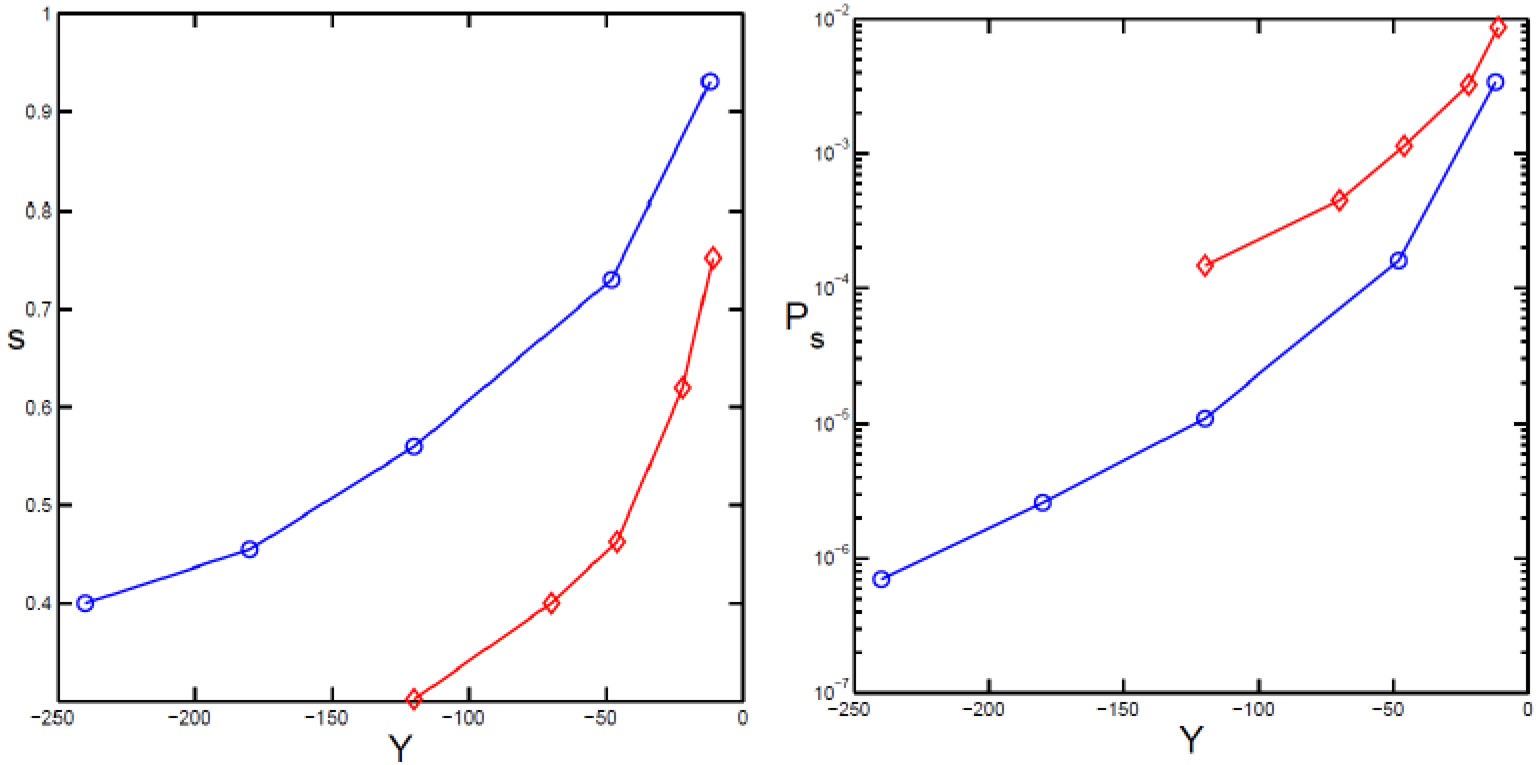
Markers show ROMA scaling using S(Y) and P_s(Y).

Comparison of $s(Y)$ and $P_s(Y)$: For $\delta v_{||} > 0$ (the units of $\delta v_{||} = 900/800$)



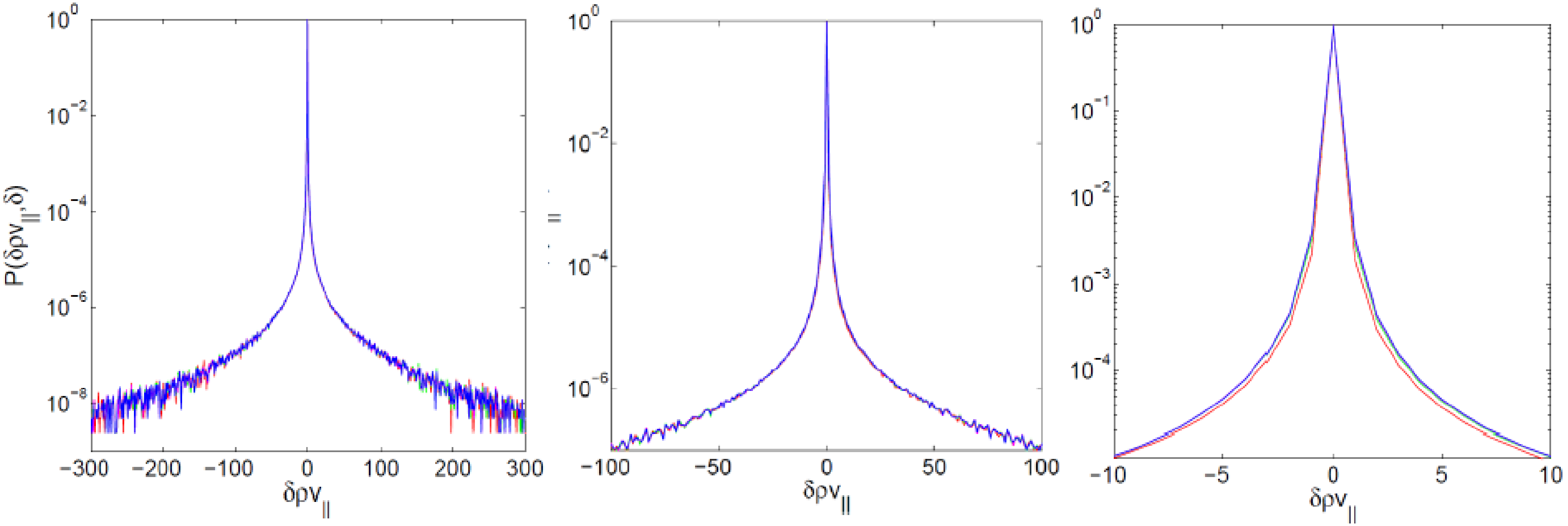
For $\delta v_{||} > 0$; blue circles for δ between 4 and 32;
Red diamonds for δ between 32 and 128.

Comparison of s(Y) and P_s(Y): For $\delta v_{||} < 0$ (the units of $\delta v_{||}$ = 900/800)



For $\delta v_{||} < 0$; blue circles for δ between 4 and 16;
Red diamonds for δ between 32 and 56.

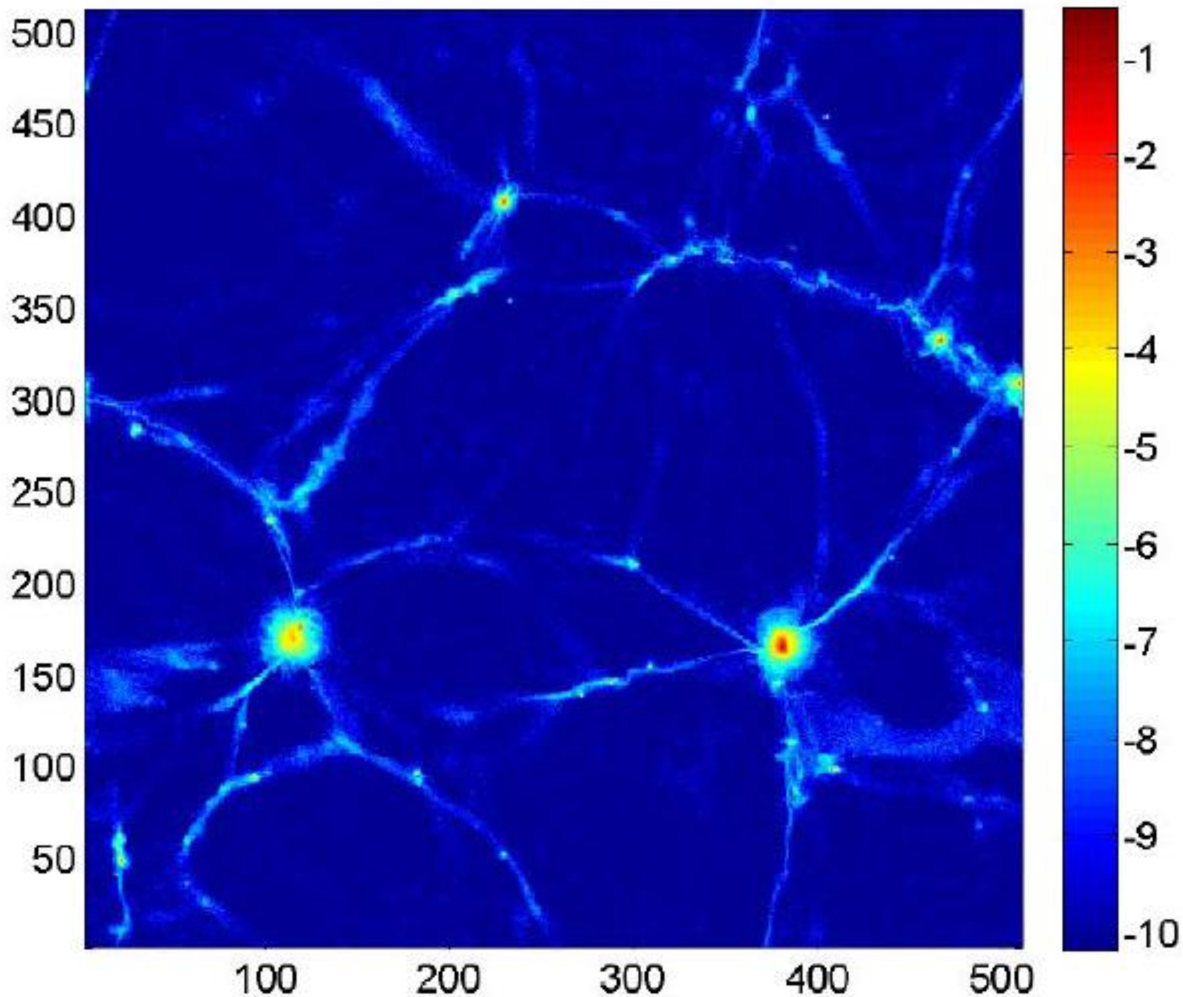
(V) PDF of $\rho v_{||}$ fluctuations at δ=4-128Δ with units of $\delta\ \rho v_{||}$ = 150/800.



δ=4Δ : red; δ=16Δ : green; δ=64Δ : magenta; δ=128Δ : blue

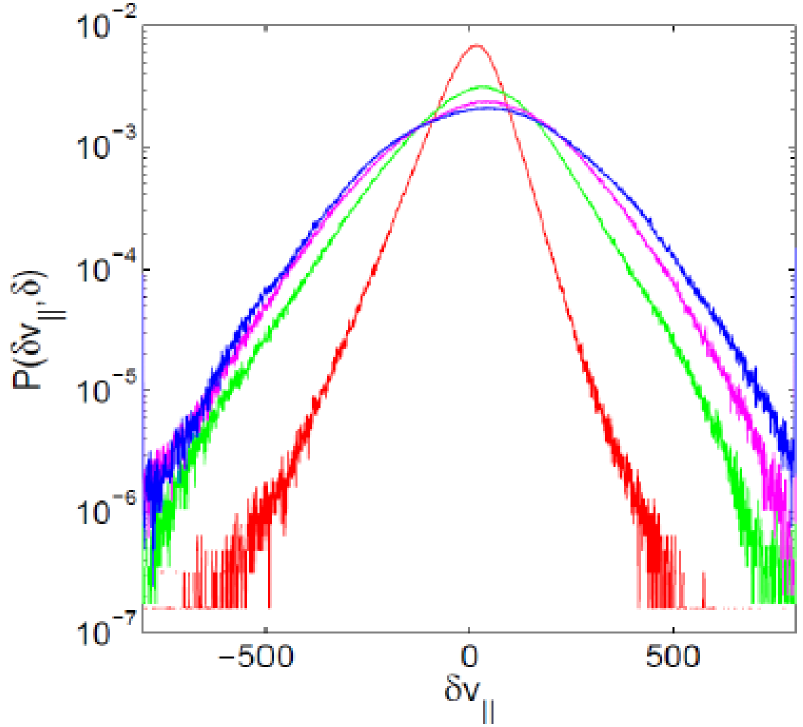
Gas particles in 512^3 cells

Density distributions at $0<x<\Delta$, with $\Delta=20/512$ Mpc/h



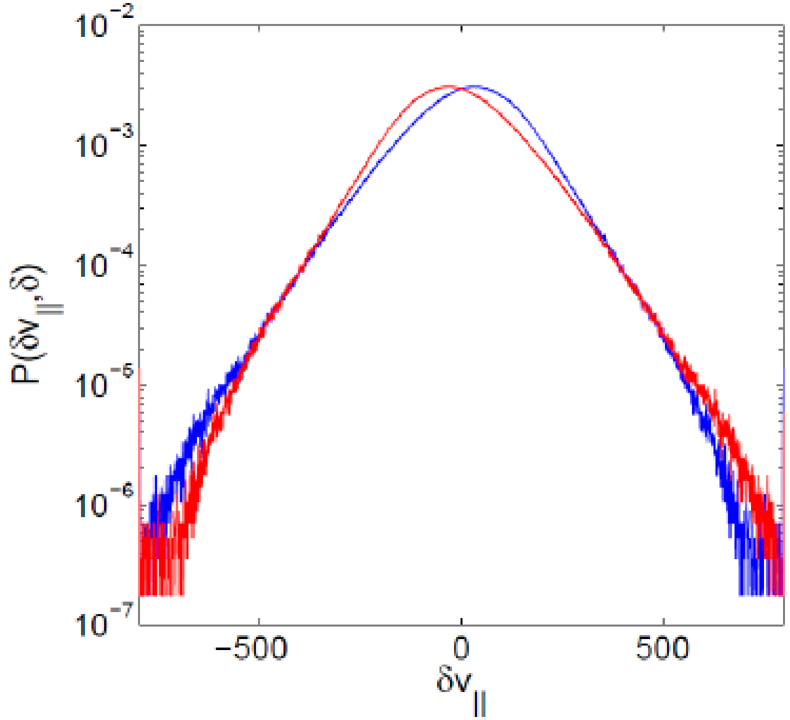
ln(ρ) as function of (y, z);

PDF $(\delta v_{||},\delta)$ in Region 1, units of $\delta v_{||}$=600/800:



Red ($\delta=4\Delta$); green ($\delta=16\Delta$);
Magenta ($\delta=32\Delta$); blue ($\delta=48\Delta$);

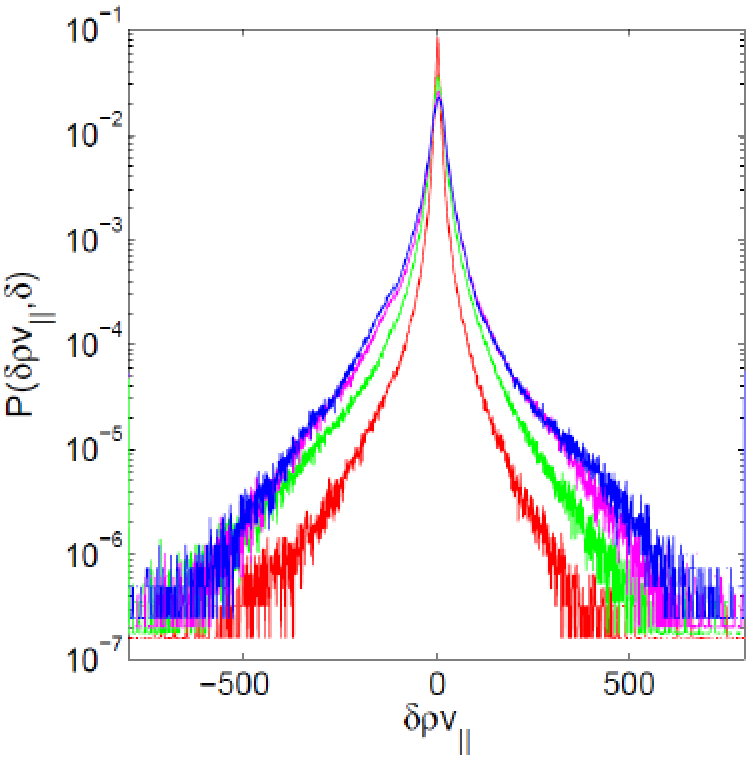
Asymmetry wrt $\delta v_{||}=0$



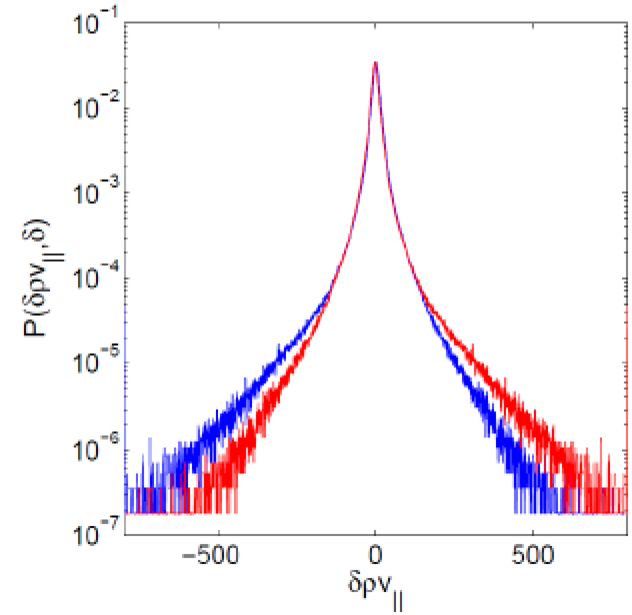
Blue: PDF$(\delta v_{||},\delta=16\Delta)$;
Red: PDF$(-\delta v_{||},\delta=16\Delta)$.

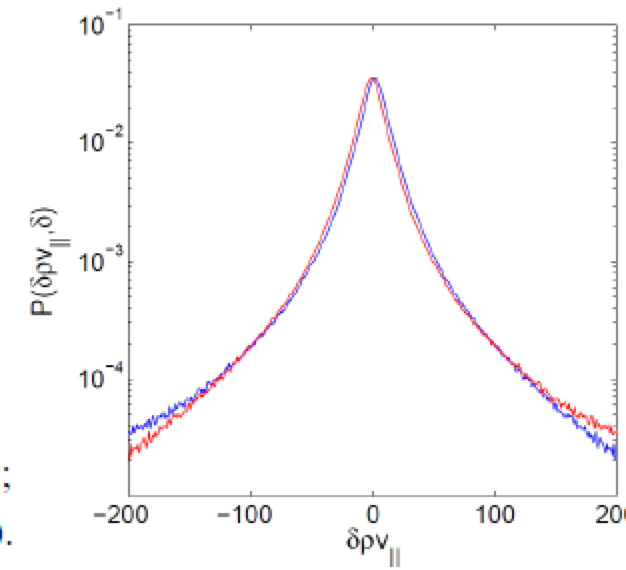
Symmetry property wrt $\delta\rho v_{||}=0$

PDF $(\delta\rho v_{||},\delta)$ in Region 1, units of $\delta\rho v_{||}=0.25/800$:



Red ($\delta=4\Delta$); green ($\delta=16\Delta$);
Magenta ($\delta=32\Delta$); blue ($\delta=48\Delta$);





Blue: PDF$(\delta\rho v_{||},\delta=16\Delta)$;
Red: PDF$(-\delta\rho v_{||},\delta=16\Delta)$.

ROMA

A LOCAL INVARIANT THEORY OF

-- CROSSOVER PHENOMENON --

OF INTERWOVEN

MULTIPLES OF FRACTALS

ROMA:
Chang and Wu, Phys. Rev. E, 77, 045401(R), 2008
Wu and Chang, Nonl. Proc. Geophys., 18, 261, 2011
Tam et al., Phys. Rev. E, 81, 036414, 2010
Chang, et al., Nonl. Proc. Geophys., 17, 545, 2010
All others: Check Google Search on ROMA
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