Plasmoid Instability in High Lundquist Number Magnetic Reconnection

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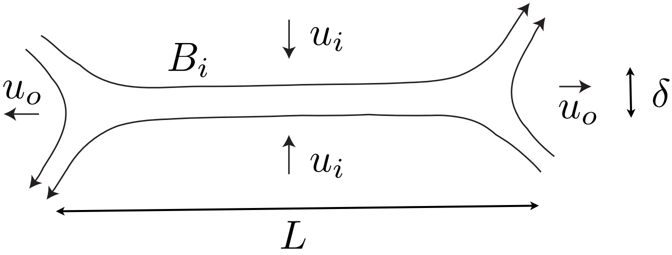
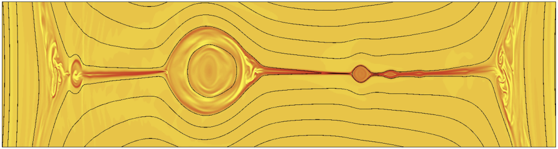
Classical Sweet-Parker Theory



- $S = LV_A/\eta$
- $\delta \sim L/\sqrt{S}$, $u_o \sim V_A$,$u_i \sim V_A/\sqrt{S}$
- Solar Corona: $S \sim 10^{12}$, $\tau_A = L/V_A \sim 1s \Rightarrow \tau_{SP} \sim 10^6 s \gg$ Solar flare time scales $10^2 - 10^3 s$.

Plasmoid Instability Leads to a Reconsideration of Fast Reconnection in Resistive MHD



- Sweet-Parker theory assumes a stable current sheet.
- Earlier simulations showed plasmoid formation in high S (Bulanov *et al.* 1979, Lee and Fu 1986, Biskamp 1986, Matthaeus and Lamkin 1986, Yan *et al.* 1992, Shibata and Tanuma 2001)
- Linear theory predicts $\gamma\tau_A \sim S^{1/4}$ and the number of plasmoids $\sim S^{3/8}$. (Loureiro et al. 2007)
- The key point is that the equilibrium also scales with S: $\delta_{SP} \sim LS^{-1/2}$.

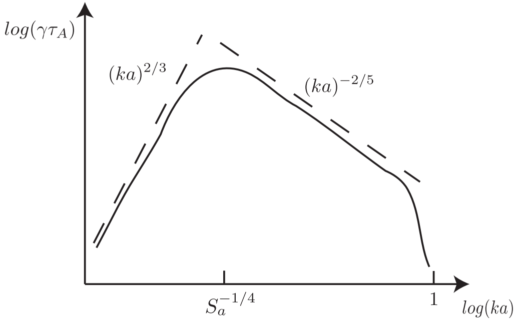
Resistive Tearing Mode Theory

Harris sheet profile $\mathbf{B} = B_o \tanh(x/a)\hat{\mathbf{y}}$

$$\gamma\tau_A \sim \begin{cases} S_a^{-3/5}(ka)^{-2/5}(1-k^2a^2)^{4/5}, & ka \gg S_a^{-1/4} \\ S_a^{-1/3}(ka)^{2/3}, & ka \ll S_a^{-1/4} \end{cases}$$

$S_a = aV_A/\eta$, $\tau_A = a/V_A$, peak $\gamma \sim S_a^{-1/2}$

Coppi et al. 1976



Resistive Plasmoid Instability

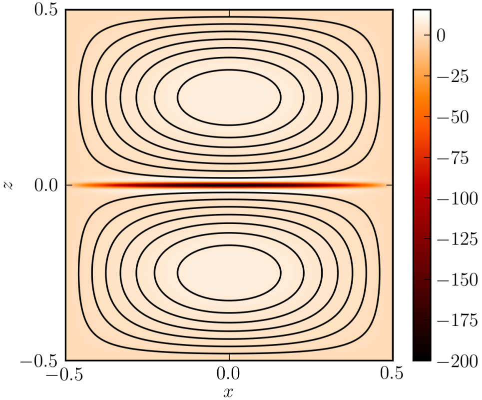
Translate to the Sweet-Parker language:
$S \equiv LV_A/\eta$, $a \to \delta_{SP} \sim LS^{-1/2}$,

$$\gamma \sim \frac{V_A}{L} \times \begin{cases} S^{2/5}\kappa^{-2/5}(1-\kappa^2\epsilon^2)^{4/5}, & \kappa \gg S^{3/8} \\ \kappa^{2/3}, & \kappa \ll S^{3/8} \end{cases}.$$

where $\kappa \equiv kL$, $\epsilon = \delta_{SP}/L$.
The peak γ occurs at $\kappa \sim S^{3/8}$ with $\gamma_{max} \sim S^{1/4}V_A/L$.

Loureiro et al. 2007, Bhattacharjee et al. 2009

Simulation Setup (Huang & Bhattacharjee 2010)



A low amplitude random forcing is added:

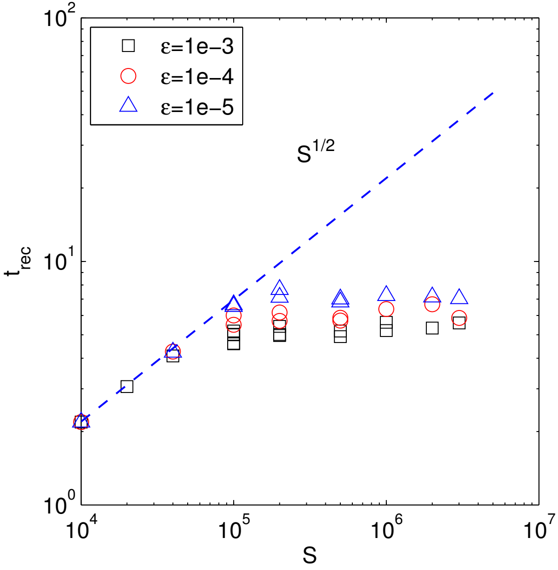
$$\partial_t(\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{u}\mathbf{u}) = -\nabla p - \nabla \psi \nabla^2 \psi + \epsilon \mathbf{f}(\mathbf{x}, t)$$

$$\langle f_i(\mathbf{x}, t) f_j(\mathbf{x}', t') \rangle \sim \delta_{ij} \delta(\mathbf{x} - \mathbf{x}') \delta(t - t')$$

Plasmoid Instability Leads to Multiple-Level Cascade

$S = 3 \times 10^6$, $\epsilon = 10^{-3}$

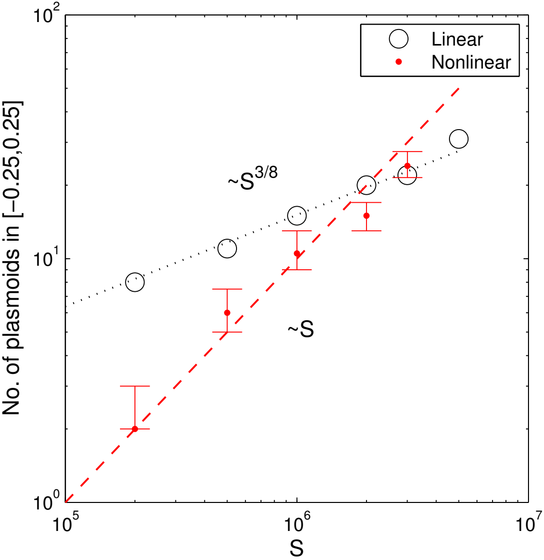
Reconnection Time of 25% of Initial Flux



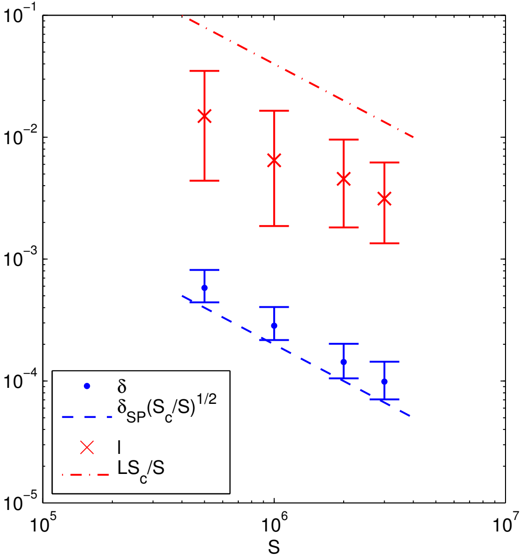
$$\left\langle \frac{1}{V_A B} \frac{d\psi}{dt} \right\rangle \sim 0.01$$

$$\langle u_i \rangle \sim 0.01 V_A$$

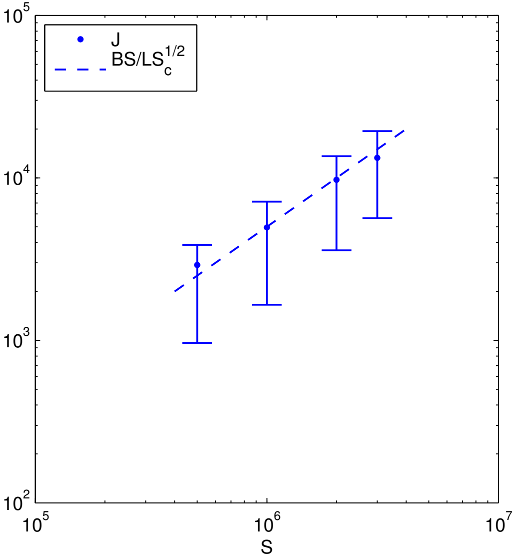
Scaling of the Number of Plasmoids



Current Sheet Width and Length $\sim 1/S$



Current Density $\sim S$



Heuristic Scaling Argument

- Cascade to smaller scales will stop when the local current sheet becomes stable to the plasmoid instability
- Consider the reconnection layer as a chain of plasmoids connected by marginally stable current sheets. For given η and V_A:
  - Critical length $L_c \sim S_c \eta / V_A \sim L S_c / S$.
  - Number of plasmoids $n_p \sim L / L_c \sim S / S_c$.
  - Current sheet with $\delta_c \sim L_c / S_c^{1/2} \sim L S_c^{1/2} / S$,
  - Current density $J \sim B / \delta_c \sim BS / L S_c^{1/2}$.
  - Reconnection rate $\sim \eta J \sim \eta B / \delta_c \sim B V_A / S_c^{1/2}$, which is independent of S.

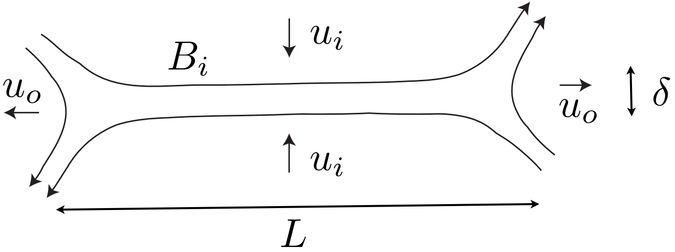
Hyper-Resistivity instead of Resistivity

$$\mathbf{E} = -\mathbf{u} \times \mathbf{B} + \eta \mathbf{J}$$
$$\Longrightarrow \mathbf{E} = -\mathbf{u} \times \mathbf{B} - \eta_H \nabla^2 \mathbf{J}$$

Hyper-resistivity has been employed from various considerations:

- (Anomalous) electron viscosity (Furth et al. 1973, Kaw et al. 1979, Aydemir 1990, Biskamp 1993, Chacon et al. 2007)
- MHD turbulence and field line stochasticity (Boozer 1986, Bhattacharjee and Hameiri 1986, Strauss 1988, Bhattacharjee and Yuan 1995)
- van Ballegooijen and Cranmer (2008) suggested energy dissipated through hyper-resistivity as an effective mechanism for coronal heating

Hyper-Resistive Sweet-Parker Theory



- The only modification is $E \sim u_i B_i \sim \eta_H B_i/\delta^3$
- $S_H = L^3 V_A/\eta_H$
- $\delta_{SP} \sim L/S_H^{1/4}$, $u_o \sim V_A$,$u_i \sim V_A/S_H^{1/4}$

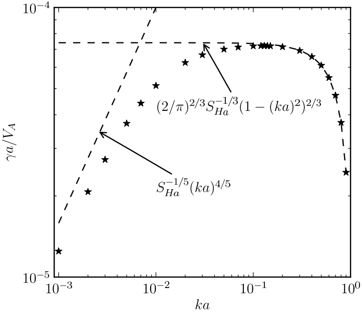
Hyper-Resistive Tearing Mode Theory

Harris sheet profile $\mathbf{B} = B_o \tanh(x/a)\hat{\mathbf{y}}$

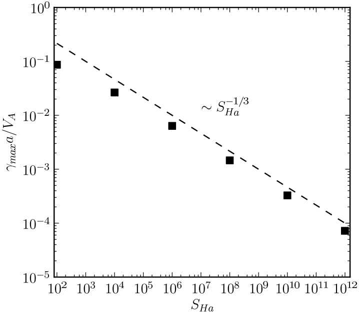
$$\gamma\tau_A \sim \begin{cases} S_{Ha}^{-1/3}(1-(ka)^2)^{2/3}, & ka \gg S_{Ha}^{-1/6} \\ S_{Ha}^{-1/5}(ka)^{4/5}, & ka \ll S_{Ha}^{-1/6} \end{cases},$$

$S_{Ha} = a^3 V_A/\eta_H$, $\tau_A = a/V_A$, peak $\gamma \sim S_{Ha}^{-1/3}$

Aydemir 1990



$S_{Ha} = 10^{12}$



Hyper-Resistive Plasmoid Instability

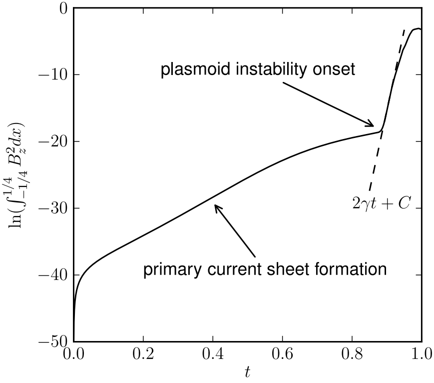
Translate to the Sweet-Parker language:
$S_H \equiv L^3 V_A/\eta_H$, $a \to \delta_{SP} \sim L S_H^{-1/4}$,

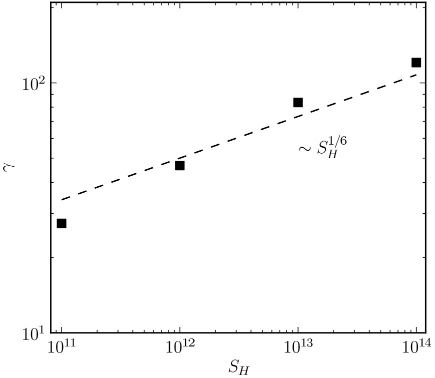
$$\gamma \sim \frac{V_A}{L} \times \begin{cases} S_H^{1/6}(1-\kappa^2\epsilon^2)^{2/3}, & \kappa \gg S_H^{5/24} \\ \kappa^{4/5}, & \kappa \ll S_H^{5/24} \end{cases} .$$

where $\kappa \equiv kL$, $\epsilon = \delta_{SP}/L$.
The peak γ occurs at $S_H^{5/24} \ll \kappa \ll S_H^{1/4}$ with $\gamma_{max} \sim S_H^{1/6} V_A/L$.

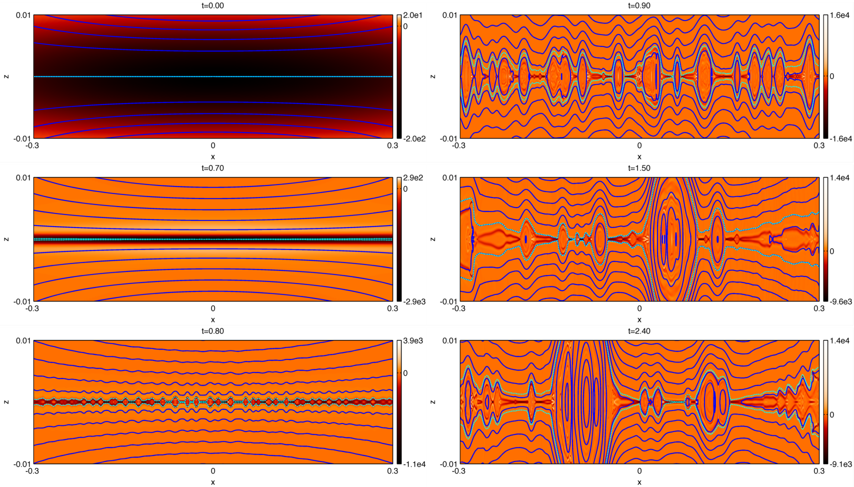
Verification of Linear Theory

Measure $g(t) \equiv \int_{-1/4}^{1/4} B_z^2(x,t)dx$ along $z = 0$.



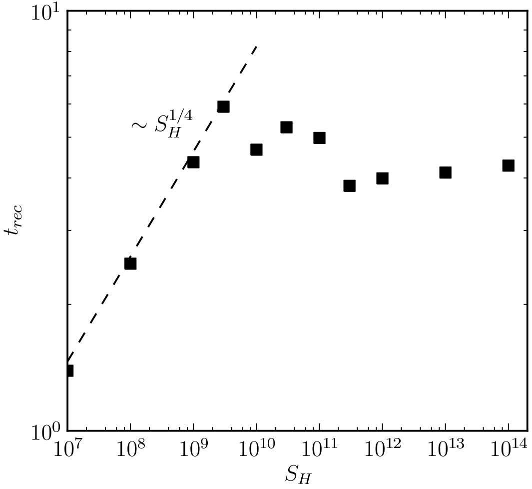


Nonlinear Evolution of the Plasmoid Instability



$S_H = 10^{14}$, $\epsilon = 10^{-4}$

Reconnection Time of 25% of Initial Flux



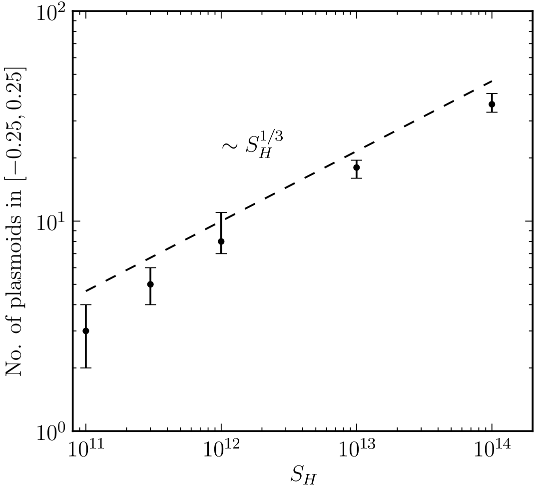
$$\left\langle \frac{1}{V_A B} \frac{d\psi}{dt} \right\rangle \sim 0.01$$

$$\langle u_i \rangle \sim 0.01 V_A$$

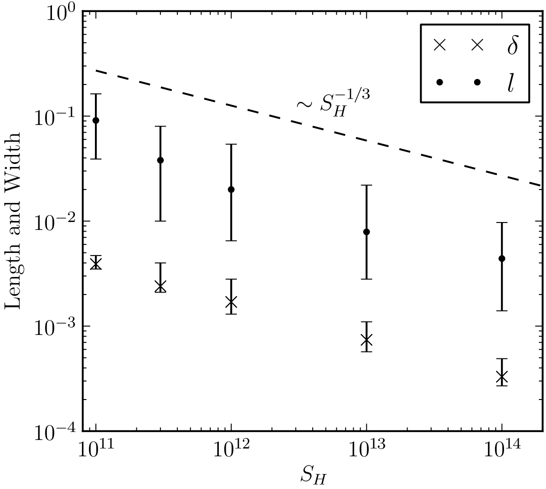
Heuristic Scaling Argument

- Consider the reconnection layer as a chain of plasmoids connected by marginally stable current sheets. For given η_H and V_A:
  - Critical length $L_c \sim (S_{Hc}\eta_H/V_A)^{1/3} \sim L(S_{Hc}/S_H)^{1/3}$.
  - Number of plasmoids $n_p \sim L/L_c \sim (S_H/S_{Hc})^{1/3}$.
  - Current sheet with $\delta_c \sim L_c/S_{Hc}^{1/4} \sim LS_{Hc}^{1/12}/S_H^{1/3}$,
  - Current density $J \sim B/\delta_c \sim (B/L)S_{Hc}^{-1/12}S_H^{1/3}$.
  - Reconnection rate $\sim \eta_H J/\delta_c^2 \sim \eta_H B/\delta_c^3 \sim BV_A/S_{Hc}^{1/4}$, which is independent of S_H.

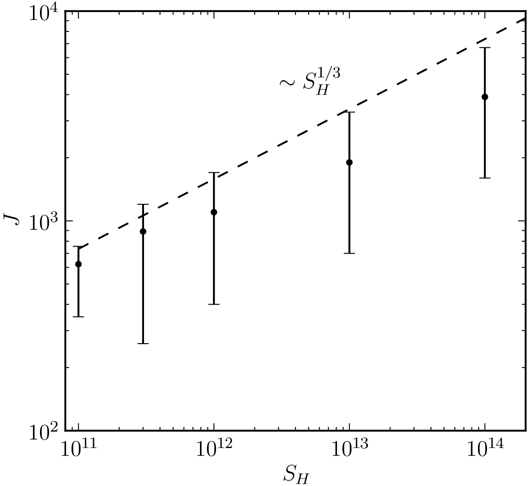
Number of Plasmoids $\sim S_H^{1/3}$



Current Sheet Width and Length $\sim S_H^{-1/3}$



Current Density $\sim S_H^{1/3}$



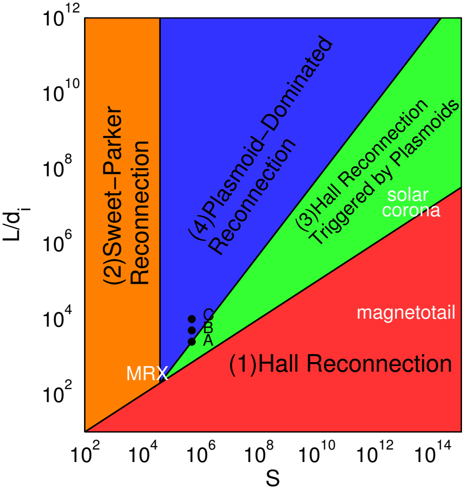
Comparison with Resistive Plasmoid Instability

Organize with respect to $\Lambda = L/\delta_{SP}$, where $\Lambda \sim S^{1/2}$ for resistive MHD, and $\Lambda \sim S_H^{1/4}$ for hyper-resistive MHD

	Resistive	Hyper-Resistive
γ_{max}	$\sim \Lambda^{1/2}$	$\sim \Lambda^{2/3}$
κ_{max}	$\sim \Lambda^{3/4}$	$\Lambda^{5/6} \ll \kappa_{max} \ll \Lambda$
n_p	$\sim \Lambda^2$	$\sim \Lambda^{4/3}$
δ and l	$\sim \Lambda^{-2}$	$\sim \Lambda^{-4/3}$
J	$\sim \Lambda^2$	$\sim \Lambda^{4/3}$
Reconnection Rate	$\simeq 10^{-2} V_A B$	$\simeq 10^{-2} V_A B$

Including the Hall Effect

- Another governing parameter L/d_i in addition to S.



A: $S = 5 \times 10^5$,
$L/d_i = 2500$
B: $S = 5 \times 10^5$,
$L/d_i = 5000$
C: $S = 5 \times 10^5$,
$L/d_i = 10000$

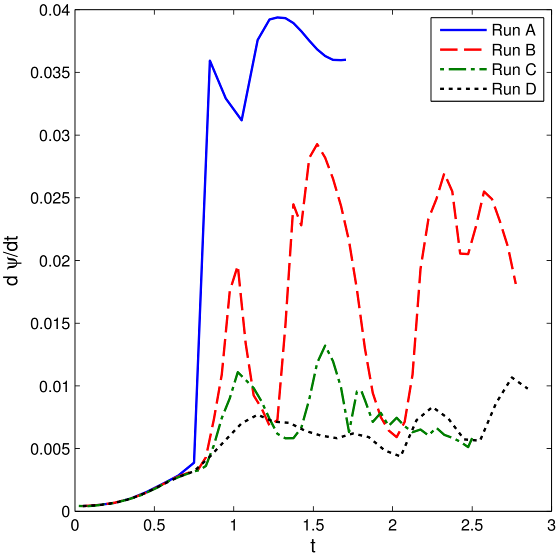
Single X-Point Hall Reconnection

$S = 5 \times 10^5,\ L/d_i = 2500$

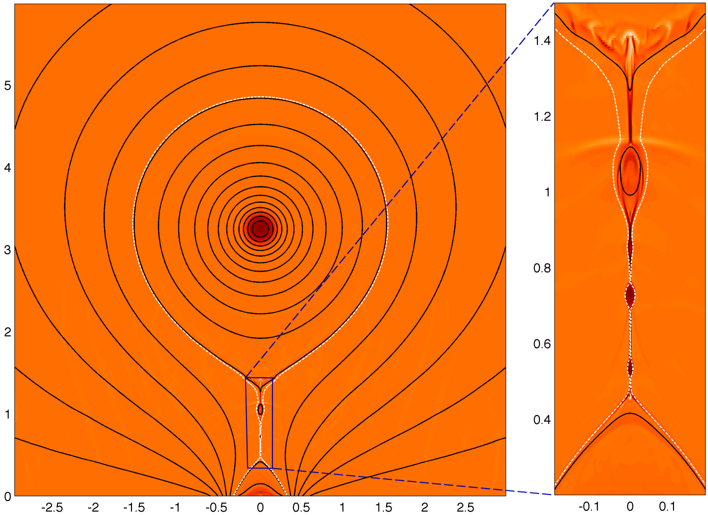
Intermediate Regime, Both S-P and Single X-Point Hall Solutions are Unstable

$S = 5 \times 10^5$, $L/d_i = 5000$

Reconnection Rate



Plasmoids in Post-CME Current Sheet



Conclusions

- Resistive and hyper-resistive plasmoid instabilities are qualitatively similar.
- Details of the scaling laws are different. However, the simple scaling argument based on marginal stability is applicable for both cases.
- In the nonlinear regime, the reconnection rate becomes nearly independent of S or S_H. The reconnection rate $\sim 10^{-2} V_A B$ for both cases.
- When the Hall effect is included, the plasmoid instability can trigger even faster Hall reconnection. However, Hall reconnection does not always settle to a single X point. There exists an intermediate regime where plasmoid formation and the Hall effect are both important.
- Plasmoid formation may be a generic feature for a broad range of fluid models with different mechanisms of breaking the frozen-in condition.